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Fibonacci Numbers and Golden Ratio: (The Divine Mathematics)

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Abstract—Fibonacci numbers are a mathematical notion that has led to the golden ratio (Ø = 1.618) and its geometric significance & application in nature is discussed in this paper. Fibonacci sequence creates the beauty in the nature & give a standard pattern for the arrangement of natural & artificial objects. The explanation demonstrates the ways in which Fibonacci numbers have existed in our society. Additionally, there are some architectural and artistic creations that affirm the existence of the golden ratio.

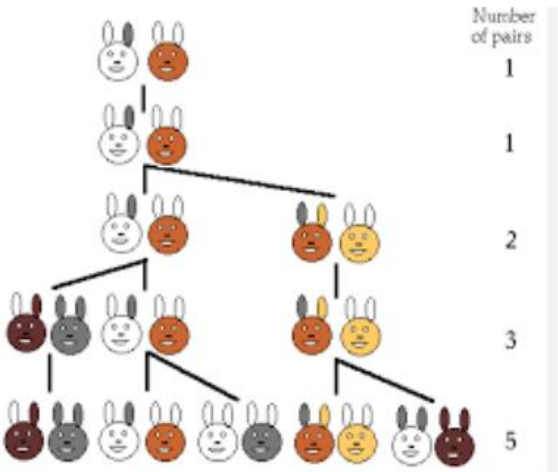
Keywords: Fibonacci Numbers; Golden Ratio; Lucas Sequence; Geometry; Spiral & Architecture.

I. INTRODUCTION

First, let's take a quick look at Leonardo Fibonacci, the most famous and acclaimed mathematician in Europe. He was born in Pisa, Italy, around 1170 into the Bonacci family, and he passed away around 1250. His real name was Leonardo of Pisa, or Leonardo Pisano; "Son of Bonacci" is what Fibonacci implies, and his father's name was Bonacci. In 1202, Leonardo of Pisa published his first book on mathematics, Liber Abaci, which was essentially on arithmetic and elementary algebra. This is the important part of the two contributions, for which he is most widely recognized.

Two more volumes, the Liber Quadratorun (Square number) and the Flos (Flowers), were produced by Fibonacci in 1225. Number theory is the subject of these books. The core issue of Rabit multipliers is where we begin.

“A man put a pair of rabbits (a male & a female) in a garden that was enclosed. How many pairs of rabbits can be produced from the original pairs within 12 months. If it is assumed that every month each pair of rabbits produce another pair (a male & a female) in which they become productive in the second month & no death, no escape of the rabbits and all female rabbits must reproduce during this period (years)"



Assume that a male and a female rabbit pair were born on 1st January. There won't be any more couples save for one on 1st February, since it will take a month before they can create another pair. We then have two pairs of rabbits on 1st March. Three pairs in 1st April, five pairs in 1st May, eight pairs on 1st June, and so forth will follow. The table that follows displays the total number of pairs.

Month	Rabit (Baby)	Rabit (Matured)	Total
1st of January	One	Zero	One
1st of February	Zero	One	One
1st of March	One	One	Two
1st of April	One	Two	Three
1st of May	Two	Three	Five
1st of June	Three	Five	Eight
1st of July	Five	Eight	Thirteen
1st of August	Eight	Fifteen	Twenty-One
1st of September	Thirteen	Twenty-One	Thirty-Four
1st of October	Twenty-One	Thirty-Four	Fifty-Five
1st of November	Thirty-Four	Fifty-Five	Eighty-Nine
1st of December	Fifty-Five	Eighty-Nine	One Hundred Forty-Four

The final column (total pairings) of the preceding table provides 1,1,2,3,5,8,13,21,34,55,89,144, or what are known as Fibonacci numbers. The French mathematician Lucas gave them their name in 1876. Additionally, we have 144 rabbit pairings in a year. Fibonacci numbers are among the most fascinating numerical sequences ever to be recorded. Every two odd numbers in the Fibonacci sequence are followed by an even number, which makes it unique. Additionally, the recursive Fibonacci sequence is frequently used by contemporary physicists and scientists. With the exception of the first two numbers, all Fibonacci numbers have the intriguing and unusual trait of being the sum of the two numbers that come right before them.

Example:

$1 + 1 = 2; \qquad 1 + 2 = 3$

$2 + 3 = 5; \qquad 3 + 5 = 8$

$5 + 8 = 13; \qquad 8 + 13 = 21$

it means any arbitrary Fibonacci number is equal to its the sum of its immediate two predecessor. If F_k is k^{th} Fibonacci number, then

$$F_k = F_{k-1} + F_{k-2}$$

For its origin, let us divide each Fibonacci number by its predecessor, and what we get

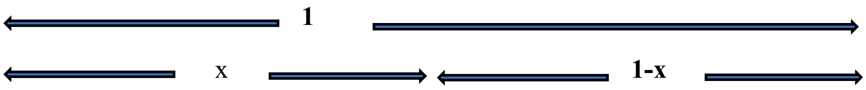
$$\frac{1}{1} = 1;\ \frac{2}{1} = 2\ ;\frac{3}{2} = 1.5;\ \frac{5}{3} = 1.666;\ \frac{8}{5} = 1.6;\ \frac{13}{8} = 1.625;\ \frac{21}{13} = 1.615;\ \frac{34}{21} = 1.619$$

$$\frac{55}{34} = 1.6176; \frac{89}{55} = 1.618;\ \frac{144}{89} = 1.618 \ldots\ \textit{and so on.}$$

We observe that the ratio approaches the ∅-function (Golden Ratio) of 1.618. The enormous pyramids were built by the Egyptians using this Golden Ratio [6]. It is represented by the Greek character phi ∅. The famous Greek mathematician Phidias, who was also a renowned sculptor, gave the letter ∅ the name phi. One of its characteristics is the partition of a line segment into two segments, a concept that arose in plane geometry.

We can refer to a ratio as the golden ratio if we split a live event so that the ratio of the length of the longer segment to the length of the shorter segment happens to be equal to the ratio of the length of the whole length to the length of the larger segment.

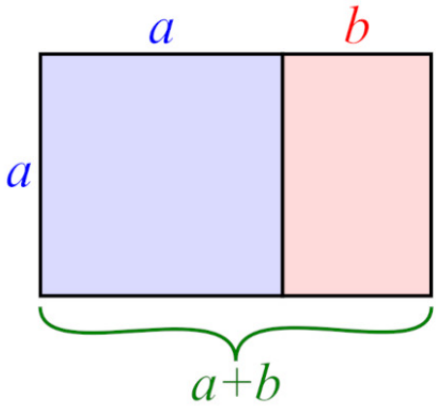
In figure shows the partition of a whole length 1 into two segments *x* and (1-*x*).



If 1/x = x/(1 - x) fulfils the quadratic equation x^2+x-1=0, the mean ratio is obtained. Since length cannot be negative, x=(-1+√5)/2=0.61803…, but we are interested in the positive roots, therefore x=(-1±√5)/2.

Hence the desired ratio is $\emptyset = \frac{1}{x} = 1.61803 \ldots$.

II. GOLDEN RECTANGLE



$$\frac{a+b}{a} = \frac{a}{b} = \emptyset \ (Golden\ Ratio) \simeq 1.6180 \ldots$$

III. GOLDEN RATIO/GOLDEN SECTION/DIVINE PROPORTION/FIBONACCI RATIO/GOLDEN NUMBER

Modern research, especially theoretical physics, makes extensive use of the irrational number, often known as the divine proportion, golden ratio, or golden mean [4]. People are curious about the numerous features of the golden ratio $\emptyset$. It is a number that, when multiplied by 1, equals the reciprocal of its own.

i.e. $\emptyset = \frac{1}{\emptyset} + 1.$

or, $\emptyset^2 - \emptyset - 1 = 0$ is the quadratic equation of golden number. Now, we know that the solution of quadratic equation is $\emptyset = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$.

But golden number is a positive number, hence we have $\emptyset = \frac{1+\sqrt{5}}{2} \simeq 1.61803 \ldots$.

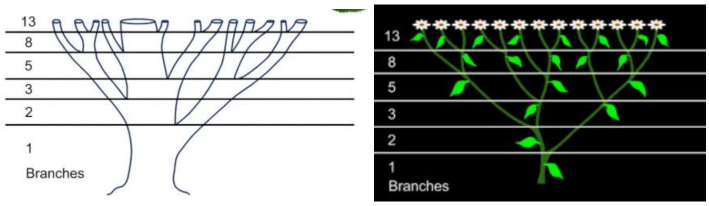
i.e. Golden Ratio $\emptyset = 1.61803 \ldots$.

IV. FIBONACCI NUMBERS AND NATURE

The arrangement of the petals on flowers, the leaves on plant stems, the scales on pineapples, or the bracts on pinecones are examples of the Fibonacci pattern found in nature. The hands, legs, face, and other parts of the human body likewise exhibit this sequence.

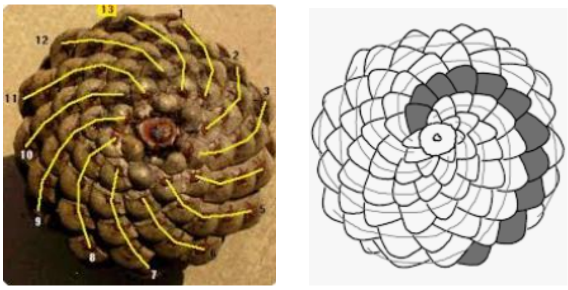
A. FIBONACCI NUMBERS IN PLANTS

Some plants exhibit this pattern in the growth of their leaves and branches. This type of design maximizes sunlight exposure while offering the best physical accommodations for the quantity of leaves and branches.



B. FIBONACCI NUMBERS IN PINEAPPLES, PINECONES, AND SUNFLOWERS

The amounts of the Fibonacci number [5] are seen in the scale pattern of the tapered pine cone, the sunflower seed pattern, and the pineapple bumps.
The pinecones have two sets of spirals: one has eight spirals that run clockwise, while the other has thirteen spirals that run counterclockwise. Nevertheless, when the spirals are joined, we can observe that this number in pinecones is found to be adjacent.



Sunflowers also have two-dimensional seeds that are arranged in a golden spiral. There are 34 spirals when counting clockwise and 21 spirals when counting counterclockwise.

The pineapples' scales have three pairs of spirals and are hexagonal in shape. The three sets are grouped as illustrated in the diagrams below.

C. FIBONACCI NUMBERS IN FLOWERS

In so many Flowers (white lily, Euphoria, Trillium, Bloodroot Shasta daisy Fibonacci sequence pattern is followed in their petals.

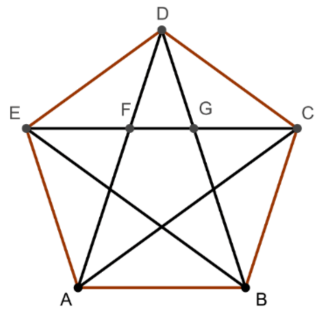
D. FIBONACCI NUMBERS IN HUMAN HAND

A close examination of our hands would reveal that, with the exception of the thumb, which is divided into two sections, each finger is divided into three segments. Five (5) fingers make up each hand, for a total of eight fingers split in three parts. All of these numbers are part of the Fibonacci sequence [2].

E. FIBONACCI NUMBERS IN HUMAN FACE

The beautifulness of face is also depending on this sequence. If the pentagonal part of face and considered the ratio of AB to CD is more nearly about golden ratio $\emptyset = 1.61803$ ... then the face cut is more attractive & beautiful.

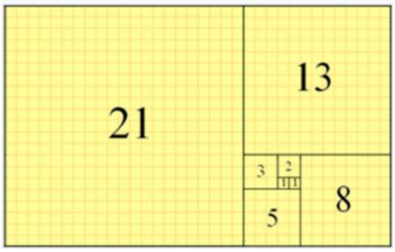
$\frac{AB}{CD} \simeq 1.61803 \ldots$.



V. FIBONACCI NUMBERS AND GEOMETRY

A. FIBONACCI RECTANGLE

Two squares of size 1 unit can be placed next to one another, and since 1+1=2, we can draw a second square of 2 units on top of the first two. Drawing a three-unit square can continue this.



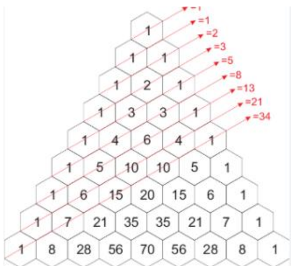
to touch a unit square and latter square of side 2 unit. Similarly, continuing process we get Golden Rectangle.

B. FIBONACCI SPIRAL

Self-similarity is a type of pattern found in nature that increases or extends in size without altering shape or staying constant. Spirals are one example of this structure. For example, the waves that brought the tide in curved into spirals that could be mathematically diagrammed at positions 1, 1, 2, 3, 5, 8, 13, 21, 34, and 55 as tides rode into the shore and currents went to the ocean. By joining the opposing corners of squares in the Fibonacci tilling with arcs composed of Fibonacci numbers, one can create a spiral. It can be seen in snail and seashell shells, the Milky Way Galaxy, and many other objects.

VI. FIBONACCI NUMBERS IN PASCAL TRIANGLE

The famous French mathematician Blaise Pascal, who was deeply interested in both mathematics and theology, devised Pascal's triangle, a triangular array [3]. where the mysterious Fibonacci number sequence can be found. These Fibonacci numbers were calculated by adding the diagonal numbers of Pascal's triangle. It wasn't until 400 years later that renowned number theorist Fibonacci realized the link between his rabbit's dilemma and his theory of probability.



VII. GOLDEN RATIO IN HUMAN

(I) IN HUMAN BODY

Numerous human body components are proportioned in accordance with the golden ratio, which is also the foundation for "Neufert," the most significant reference work for contemporary architects. The first example I want to give is that the typical human body has a height of 1.618 when the foot and navel are one inch apart. A few more instances of the golden ratio are as follows: if the wrist and elbow are one distance apart, the fingertip and elbow will be 1.618; if the jaw and top of the head are one distance apart, the shoulder line and top of the head will be 1.618. There are plenty more examples of golden ratio in human body.

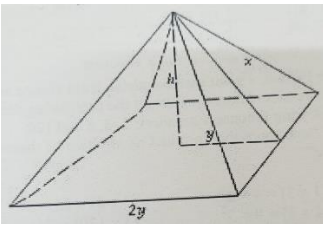
(II) GOLDEN RATIO IN HUMAN FACE

It has been demonstrated that the divine ratio is present across the human face. For instance, the distance between the lips and the point where the eyebrows meet is 1.618 over the length of the nose 1, and the length of the face is 1.618 over the breadth of the face 1. Additionally, the ratios of the nose's width (1.618) to the distance between nostrils 1 and mouth length (1.618) have been calculated.

VIII. GOLDEN RATIO IN GREAT PYRAMID

The pyramids built by the Egyptians were regarded as the most exquisite and striking structures in the world. People find the pyramids fascinating due to their enormous size and straightforward design. The great pyramid of Giza was designed by the ancient Egyptians about 4500 years ago using the golden ratio as a guide. The great pyramid's height is 484.4 feet, or roughly 5813 inches, which corresponds to three successive Fibonacci numbers: 5, 8, and 13.

According to Herodotus, a Greek historian, the Egyptian priests informed him that the large pyramid's proportions were chosen such that the "area of a square whose length of A is the same as the height of pyramid equals area of a triangular face." Herodotus' claims, according to George Mark Onsky, indicate that the "ratio of the triangular face or slant height half of the length of the base of great pyramid..." The figure illustrates it.



According to Herodotus $h^2 = \frac{2y \times x}{2} = xy$

and Pythagoras $h^2 = x^2 - y^2 \Rightarrow x^2 - y^2 = xy$

$\Longrightarrow \left(\frac{x}{y}\right)^2 - 1 = \frac{x}{y}$, this can be writing as $\left(\frac{x}{y}\right)^2 - \frac{x}{y} - 1 = 0$ which satisfies the quadratic equation $\emptyset^2 - \emptyset - 1 = 0$ where $\emptyset = \frac{x}{y}$.

According to Thomas Koshy, the great pyramid's base is 116.4 meters, its height is 148.2 meters, and its slant height is 188.4 meters. Since x/y=188.4/148.2=1.618, the golden ratio value $\emptyset$, is established.

IX. CONCLUSION AND EVALUATION

The Fibonacci number's presence in nature and its connection to the golden ratio are the primary outcome of this chapter/paper. According to research, the enigmatic Fibonacci number appears in a variety of objects, including plants, fruits, flowers, human hands, and faces.

Additionally, it is demonstrated that the recursive relation form $F_n = F_{n-1} + F_{n-2}$ with beginning conditions $F_1 = F_2 = 1$ for n ≥ 3 may be used to mathematically create the Fibonacci number. The results of this study demonstrate that the numbers in the Fibonacci series were structured so that there is an even number for every two odd numbers and that multiples of three and five varied at equal intervals. This chapter/paper explains the applications of the Fibonacci sequence in mathematics and the golden ratio in art and architecture. Through the use of the Fibonacci number and 1:1.618 to create a golden rectangle and evaluate F_{n+1}/F_n, this study aimed to demonstrate the relationship between the Fibonacci number and the golden ratio. As the value of n increases, the outcome yields $\emptyset$. The Fibonacci number's presence in Pascal's triangle is another discovery that demonstrates the application of the golden ratio in the building of the Egyptian Great Pyramid of Giza.

X. LIMITATION OF RESEARCH

The primary and most important limitations of this study are the conclusions that the human face and body are proportionate to the golden ratio. There is no real measurement of the human body or face in this research, so the findings are based on the literature of the experts.

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