Soret-Dufour effect on Unsteady MHD flow of Dusty viscoelastic fluid over inclined porous plate embedded in porous medium

Ravi Kant Yadav[1],

[1]Department of Applied Science, Babu Banarasi Das Institute of Technology and Management, Lucknow-226028, India

*ravikant.yadav31@gmail.com

Abstract In the current work, a continuously moving inclined plate with the Soret-Dufour effect is subjected to an unstable two-dimensional MHD (Magnetohydro Dynamics) flow of a dusty viscoelastic (Walter's liquid model-B) incompressible viscous and electrically conducting fluid. The coupled equations involving a nonlinear problem are investigated using the Crank-Nicolson finite difference method, and a numerical solution for the velocity, temperature, and concentration distributions is achieved.
The graphical findings are shown to interpret the impact of the problem's many significant parameters. The skin-friction, heat, and mass transfer coefficients at the plate walls are tabulated as well as their effects.
Keywords: MHD, heat transfer, Finite difference method.

I Introduction

Due to their use in geophysics, particularly in the study of the Earth's core, where the Earth's mantle, which is composed of conducting fluid, behaves like a porous medium and can become convectively unstable as a result of differential diffusion, the subject of the effect of magnetic field on thermal instability of Walter's B model elasticoviscous fluid has attracted the attention of a number of scholars recently.
Nield and Bejancite [1] have written a thorough assessment of the research on the convective heat transfer mechanism through porous media. In the presence of viscous dissipation and ohmic heating, Osalusi et al. [2] studied the effects of thermo-diffusion and diffusion-thermo on the combined heat and mass transport of a steady hydromagnetic convective and slip flow caused

by a revolving disc. In the presence of changing viscosity impact, Kafoussias and Williams [3] studied the mixed forced convection boundary layer flow with the effects of thermal diffusion and diffusion thermo. Gireesha et al. [4] provided a numerical solution for the MHD flow and heat transfer of a dusty fluid across a linearly extending sheet.
The effects of chemical reactions on the MHD flow of a dusty viscoelastic (Walter's liquid model B) liquid with a heat source/sink have been studied by Kumar and Srivastava [6]. Khan et al. [8] have reported unstable free convection flow in the Walters' B fluid and heat transfer study (reference Khan). Effects of Thermophoresis, Dufour, Hall, and Radiation on an Unsteady MHD flow past an Inclined Plate with Viscous Dissipation, N. Pandya and A. K. Shukla, [9]. The effects of Soret-Dufour and radiation on an unsteady MHD flow across an inclined porous plate embedded in a porous medium with viscous dissipation have been addressed by N. Pandya and A. K. Shukla [10].The effects of the dusty viscous fluid on turbulent free convective flow through a porous hot vertical plate with thermal diffusion and mass transfer solved by perturbation techniques have been studied by Dubey et al. [5]. Effect of Dusty Viscous Fluid on Unsteady Laminar Free Convective Flow through Porous Medium Along a Moving Porous Hot Vertical Plate with Thermal Diffusion was recently explored by Dubey et al. [7].Mishra et al.[11] studied Control of dusty nanofluid due to the interaction on dust particles in a conducting medium.Dusty nanofluid flow with bioconvection past a vertical stretching surface had been analised by Dey et al.[12].Mahanthesh et al.[13] provided Significance of quadratic thermal radiation and quadratic convection on boundary layer two-phase flow of a dusty nanoliquid past a vertical plate
This work discusses the Soret-Dufour effect on the turbulent MHD flow of a dusty viscoelastic fluid past an inclined porous plate immersed in a porous medium with temperature and mass diffusion variations. The Crank-Nicolson implicit finite difference approach is used to solve the problem's set of governing equations and boundary equation, which are converted into a set of nonlinear partial differential equations. On the velocity, temperature, and concentration profiles as well as the Skin-friction, local Nusselt, and Sherwood values are the effects of various physical parameters discussed.

II Mathematical Analysis

The flow of an infinitely inclined plate past an incompressible dusty viscoelastic fluid with changing temperature and mass diffusion has been examined in an unsteady MHD flow. The plate is set in a porous material and tilted at an angle of λ to vertical. The plate has been taken together with the x' and y' axes, respectively. The y'-axis is assumed to have a constant magnetic field, $B0$, and the plate is electrically non-conducting. Due to the low magnetic Reynolds number and transversely applied magnetic field, Cowling [14] that the induced magnetic field is insignificant in contrast to the applied magnetic field.
Due to the infinite length in the x' direction, the only variables that affect flow are t' and y'. Take into account the standard Boussinesq approximation, the governing equations for the flow field are:

$$\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = -v_0(constant) \tag{1}$$

$$\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}=\nu\frac{\partial^2 u'}{\partial y'^2}-k_0\frac{\partial^3 u'}{\partial y'^2\partial t'}+g\beta(T'-T'_\infty)\,cos(\lambda)+g\beta^*(C'-C'_\infty)\,cos(\lambda)+\frac{KN_0}{\rho}(V-u')-\frac{\sigma B_0^2}{\rho}u'-\frac{\nu u'}{K'} \tag{2}$$

$$m_1\frac{\partial V}{\partial t'}=K(u'-V) \tag{3}$$

$$\rho C_p\left(\frac{\partial T'}{\partial t'}+v'\frac{\partial T'}{\partial y'}\right)=k\frac{\partial^2 T'}{\partial y'^2}+\frac{\rho D_m K_T}{c_s}\frac{\partial^2 C'}{\partial y'^2} \tag{4}$$

$$\frac{\partial C'}{\partial t'}+v'\frac{\partial C'}{\partial y'}=D_m\frac{\partial^2 C'}{\partial y'^2}+\frac{D_m K_T}{T_m}\frac{\partial^2 T'}{\partial y'^2} \tag{5}$$

where β^* is coefficient of volume expansion for mass transfer, β is volumetric coefficient of thermal expansion, v' is velocity along y'-axis, K' is permeability of porous medium, σ is electrical conductivity, k_0 Walter's-B viscoelasticity parameter, D_m is molecular diffusivity, g is acceleration due to gravity, K_T is thermal diffusion ratio, V is the velocity of dust particles, K is the stoke's resistance coefficient, N_0 is the number density of the dust particles which is taken to be constant, m_1 is the mass of dust particles, ρ is fluid density, k is thermal conductivity of fluid, C' and T' are dimensional concentration and temperature, C'_∞ and T'_∞ are concentration and temperature of free stream, ν is kinematic viscosity and T_m is mean fluid temperature.
Boundary and initial conditional for this model are given as:

$$\begin{aligned}
t'\le 0 \quad u'=0 \quad T'=T'_\infty \quad C'=C'_\infty \quad \forall y'\\
t'>0 \quad u'=u_0 \quad v'=-v_0 \quad T'=T'_\infty+(T'_w-T'_\infty)e^{-At'},\\
C'=C'_\infty+(C'_w-C'_\infty)e^{-At'} \quad at\,y'=0\\
u'=0 \quad T'\to T'_\infty \quad C'\to C'_\infty \quad y'\to\infty
\end{aligned} \tag{6}$$

where T'_w and C'_w are concentration and temperature respectively of plate and $A=\frac{v_0^2}{\nu}$.

we introduce following quantities to obtain non-dimensional form of governing equations,

$$\begin{aligned}
u=\frac{u'}{u_0},\ t=\frac{t'v_0^2}{\nu},\ \theta=\frac{T'-T'_\infty}{T'_w-T'_\infty},\ C=\frac{C'-C'_\infty}{C'_w-C'_\infty},\ Gm=\frac{\nu g\beta^*(C'_w-C'_\infty)}{u_0v_0^2},\\
Gr=\frac{\nu g\beta(T'_w-T'_\infty)}{u_0v_0^2},\ Du=\frac{D_mK_T(C'_w-C'_\infty)}{c_sc_p\nu(T'_w-T'_\infty)},\ Sr=\frac{D_mK_T(T'_w-T'_\infty)}{T_m\nu(C'_w-C'_\infty)},\\
K=\frac{v_0^2K'}{\nu^2,},\ Pr=\frac{\mu c_p}{k},\ M=\frac{\sigma B_0^2\nu}{\rho v_0^2},\ Sc=\frac{\nu}{D_m},\ y=\frac{y'v_0}{\nu},\ \Gamma=\frac{k_0v_0^2}{\nu^2}\\
v=\frac{V}{u_0},\ B_1=\frac{\nu KN_0}{\rho u_0^2},\ B=\frac{m_1u_0^2}{VK}
\end{aligned} \tag{7}$$

Using the above non dimensional quantities, the equations 2, 3 and 5 in the non-dimensional form can be written as

$$\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-\Gamma\frac{\partial^3 u}{\partial y^2\partial t}+Gr\,cos(\lambda)\,\theta+Gm\,cos(\lambda)\,C+B_1(v-u)-\left(M+\frac{1}{K}\right)u \tag{8}$$

$$B\frac{\partial v}{\partial t} = u - v \tag{9}$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} + Du\frac{\partial^2 C}{\partial y^2} \tag{10}$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} + Sr\frac{\partial^2 \theta}{\partial y^2} \tag{11}$$

and corresponding initial and boundary conditions are

$$\begin{aligned} t \leq 0 \quad u = 0 \quad \theta = 0 \quad C = 0 \quad \forall\, y \\ t > 0 \quad u = 1 \quad \theta = e^{-t} \quad C = e^{-t} \quad at\; y = 0 \\ u = 0 \quad \theta \to 0 \quad C \to 0 \quad y \to \infty \end{aligned} \tag{12}$$

It is now important to calculate the physical quantities of primary interest, which are the local wall shear stress or skin-friction coefficients τ along plate x-axis, the local surface heat or Nusselt number Nu and mass flux or Sherwood number Sh. Non-dimensional form of these physical quantities are

$$\begin{aligned} \tau &= \left(\frac{\partial u}{\partial y}\right)_{y=0} \\ Nu &= -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} \\ Sh &= -\left(\frac{\partial C}{\partial y}\right)_{y=0} \end{aligned} \tag{13}$$

III Method of Solution

Equations 8 to 11 are coupled nonlinear partial differential equations and are solved by using initial and boundary conditions 12. However, exact or approximate solutions are not possible for this set of equations. And hence we solve these equations by an implicit finite difference method of Crank – Nicolson type for a numerical solution. The equivalent finite difference scheme of equations 8, 9, 10 and 11 are expressed as

$$\begin{aligned} \frac{u_{i,j+1} - u_{i,j}}{\Delta t} - \frac{u_{i+1,j} - u_{i,j}}{\Delta y} = \\ \left(\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j} + u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1}}{2(\Delta y)^2}\right) \\ -\Gamma\left(\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j} + u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1}}{2(\Delta y)^2 \Delta t}\right) \\ +Gr\,cos(\lambda)\left(\frac{\theta_{i,j+1} + \theta_{i,j}}{2}\right) + Gm\,cos(\lambda)\left(\frac{C_{i,j+1} + C_{i,j}}{2}\right) \\ +B_1\left(\left(\frac{v_{i,j+1} + v_{i,j}}{2}\right) - \left(\frac{u_{i,j+1} + u_{i,j}}{2}\right)\right) \\ -\left(M + \frac{1}{K}\right)\left(\frac{u_{i,j+1} + u_{i,j}}{2}\right) \end{aligned} \tag{14}$$

$$B\frac{v_{i,j+1} - v_{i,j}}{\Delta t} = \left(\left(\frac{u_{i,j+1} + u_{i,j}}{2}\right) - \left(\frac{v_{i,j+1} + v_{i,j}}{2}\right)\right) \tag{15}$$

$$\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t} - \frac{\theta_{i+1,j} - \theta_{i,j}}{\Delta y} = \frac{1}{Pr}\left(\frac{\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i+1,j} + \theta_{i-1,j+1} - 2\theta_{i,j+1} + \theta_{i+1,j+1}}{2(\Delta y)^2}\right) + Du\left(\frac{C_{i-1,j} - 2C_{i,j} + C_{i+1,j} + C_{i-1,j+1} - 2C_{i,j+1} + C_{i+1,j+1}}{2(\Delta y)^2}\right) \tag{16}$$

$$\frac{C_{i,j+1} - C_{i,j}}{\Delta t} - \frac{C_{i+1,j} - C_{i,j}}{\Delta y} = \frac{1}{Sc}\left(\frac{C_{i-1,j} - 2C_{i,j} + C_{i+1,j} + C_{i-1,j+1} - 2C_{i,j+1} + C_{i+1,j+1}}{2(\Delta y)^2}\right) + Sr\left(\frac{\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i+1,j} + \theta_{i-1,j+1} - 2\theta_{i,j+1} + \theta_{i+1,j+1}}{2(\Delta y)^2}\right) \tag{17}$$

boundary and initial conditions are also expressed as:

$$\begin{aligned} u_{i,0} = 0 \quad \theta_{i,0} = 0 \quad C_{i,0} = 0 \quad \forall\, i \\ u_{0,j} = 1 \quad \theta_{0,j} = e^{-j\Delta t} \quad C_{0,j} = e^{-j\Delta t} \\ u_{N,j} = 0 \quad \theta_{N,j} \to 0 \quad C_{N,j} \to 0 \end{aligned} \tag{18}$$

here index i and j refer to y and time t respectively, $\Delta t = t_{j+1} - t_j$ and $\Delta y = y_{i+1} - y_i$. Known values of u, θ and C at t, we have solved above equations for values $t + \Delta t$ as follows: We obtain these values to substitute $i = 1, 2, 3, \cdots, N-1$, where N pertains to ∞ then equations 14 to 17 give tridiagonal system of equations with initial and boundary conditions in equation 18 are solved using Thomos algorithm as discussed in Carnahan et al.[15], we have been found values of θ and C for all values of y at $t + \Delta t$. Equation 14 and 15 are solved by same to substitute these values of θ and C, we get solution for u and w till desired time t. calculation were execute for $\Delta y = 0.1$, $\Delta t = 0.001$ and repeated till $y = 4$.

IV Result and Discussion

The present investigation highlights the behaviour of dusty viscoelastic (Walters' liquid model-B) stratified fluid subject to a variable temperature and mass diffusion. The system of momentum and heat transfer equation are solved numerically and analysed using thermal Grashof number Gr, Thermophoresis or Soret number Sr, Dufour number Du, solutal Grashof number Gm, viscoelastic parameter Γ, Schmidt number Sc, magnetic parameter M, dusty fluid parameter B_1, dusty particle parameter B, permeability of porous medium K, prandtl number Pr, and inclination angle λ with help of graphs.

Figures 7 elucidates the velocity of the fluid with increasing Dufour number. It is observed that velocity increases slowly as Dufour number increases. Figures 8, 11, 13 and 10 are plotted for velocity of the fluid. Increasing values of Schmidt number, dusty fluid parameter, inclination angle and dusty particle parameter decreases the velocity of fluid. Figures9, 6 and 4 display the effects of the Soret number on the velocity, temperature and concentration profiles. It is observed that, when Sr increases, the velocity and conetration increases to a great extent while first temperature decreases near the plate after that starts to increase. Figure 5 and 2 reveal that the temperature decreases slowly near the plate and after that starts to increase slowly on the otherhand concentration increases slowly near the plate and after that starts to decreases rapidly. From figure 3, it is seen that Du has remarkable effect on temperature profiles; quantitatively when Du increases from there is no change in temperature near the plate after that temperature increases slowly. The Dufour number has a falling effect on the concentration field shown in figure 1. Figure 12 illustrates the velocity field for different values of viscoelastic parameter Γ. The velocity increases near the wall rapidly after that decreases. Figures 14, 15 and 16 are plotted for velocity, temperature and concentration of the fluid. Increasing values of time t velocity increases rapidly, temperature and concentration near the plate decrease after that increase. Finally, the effects of various parameters on the skin friction τ, local Nusselt number Nu and local Sherwood number Sh are shown in the tables 1 and 3.

V Conclusion

From the present investigation we can draw the following conclusions:

1. Using Soret number we can control the heat and mass transfer flow characteristics.
2. The effect of viscoelastic parameter is very noteworth for velocity field.
3. The Dufour effect is significant
4. The dusty fluid parameter and dusty particle parameter play an important role in velocity field.

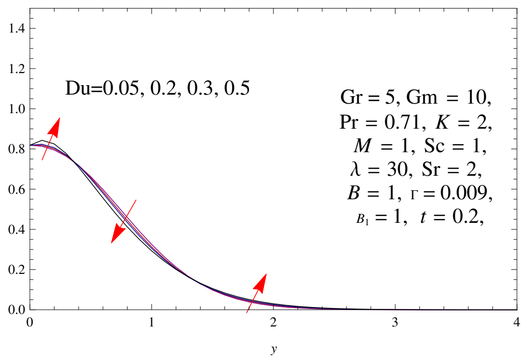

Figure 1. Concentration Profile for Different Values of Du

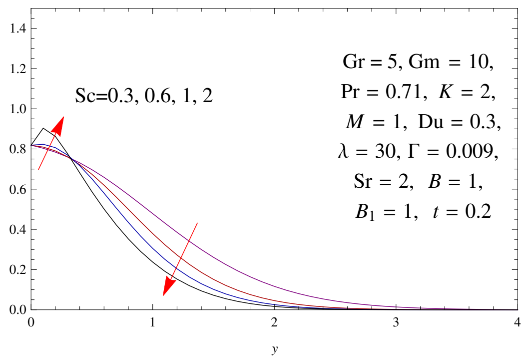

Figure 2. Concentration Profile for Different Values of Sc

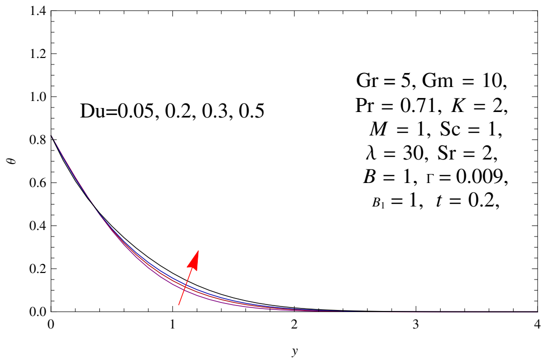

Figure 3. Temperature Profile for Different Values of Du

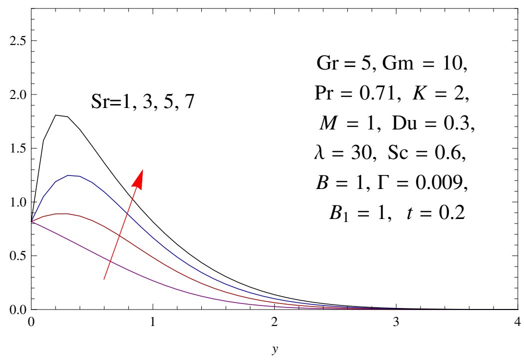

Figure 4. Concentration Profile for Different Values of Sr

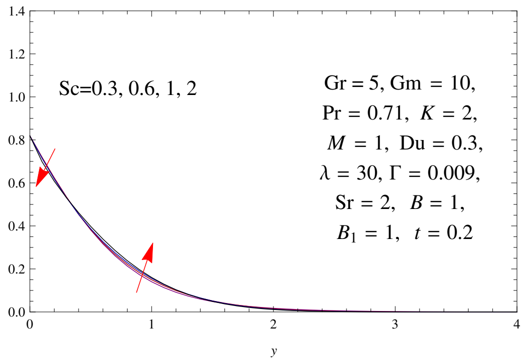

Figure 5. Temperature profile for Different Values of Sc

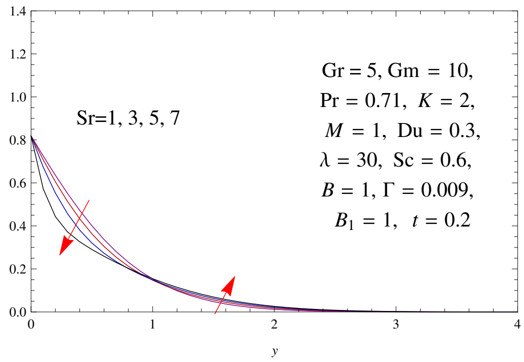

Figure 6. Temperature Profile for Different Values of Sr

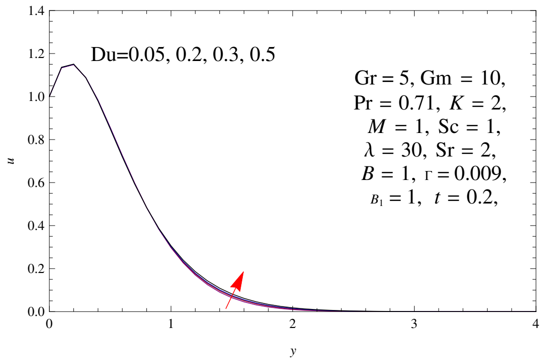

Figure 7. Velocity Profile for Different Values of Du

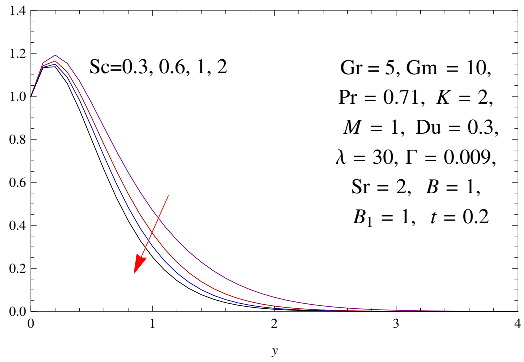

Figure 8. Velocity Profile for Different Values of Sc

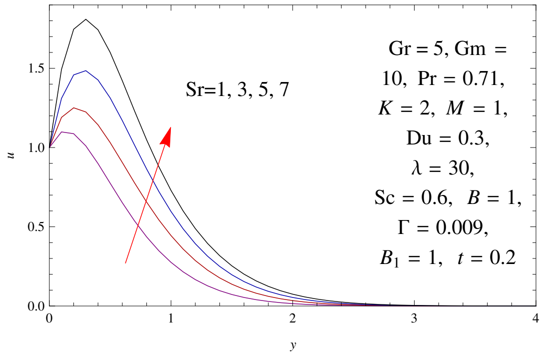

Figure 9. Velocity Profile for Different Values of Sr

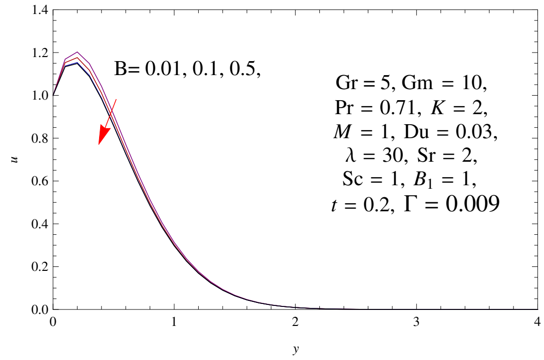

Figure 10. Velocity Profile for Different Values of B

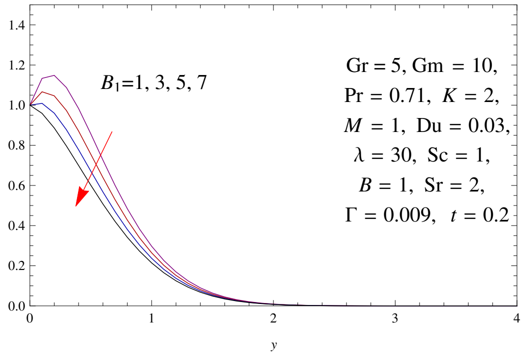

Figure 11. Velocity Profile for Different Values of B_1

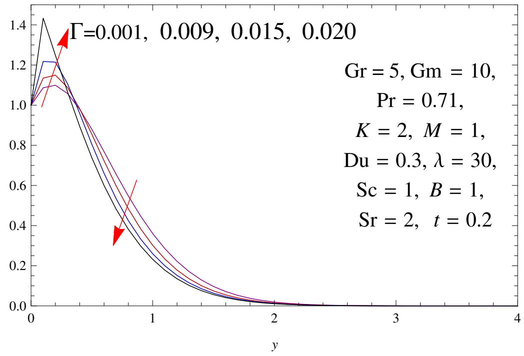

Figure 12. Velocity Profile for Different Values of Γ

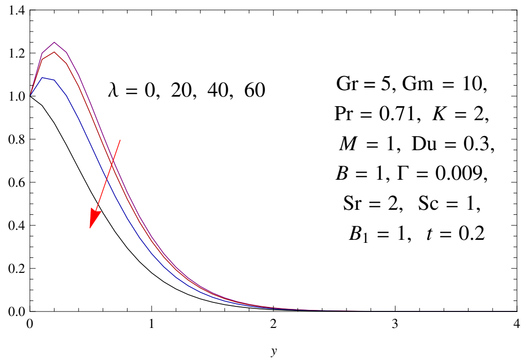

Figure 13. Velocity Profile for Different Values of λ

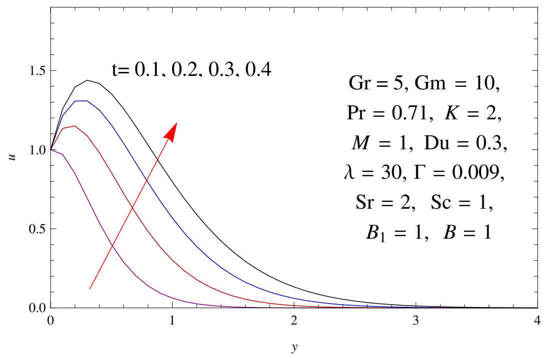

Figure 14. Velocity Profile for Different Values of t

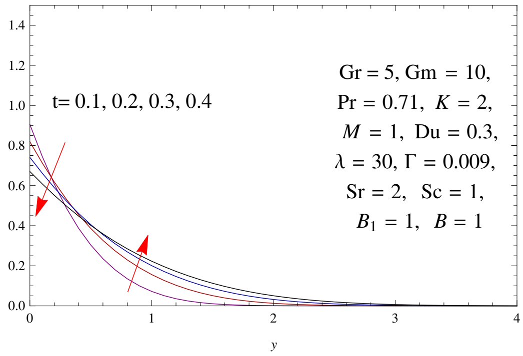

Figure 15. Temperature Profile for Different Values of t

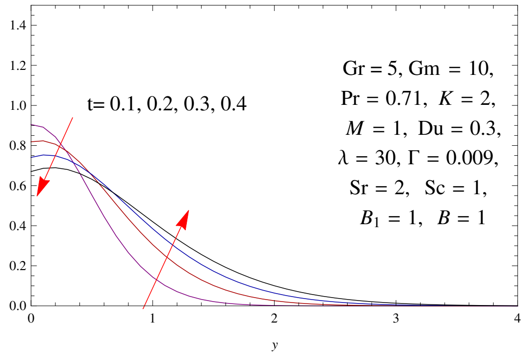

Figure 16. Concentration Profile for Different Values of t

Table 1. Skin friction coefficient τ for different values of parameters

B	B_1	Du	λ	Sc	Sr	t	Γ	τ
0.01	1	0.03	30	1	2	0.2	0.009	1.69062
0.1	1	0.03	30	1	2	0.2	0.009	1.52994
0.5	1	0.03	30	1	2	0.2	0.009	1.36899
1	1	0.03	30	1	2	0.2	0.009	1.33553
1	3	0.03	30	1	2	0.2	0.009	0.666538
1	5	0.03	30	1	2	0.2	0.009	0.0911493
1	1	0.05	30	1	2	0.2	0.009	1.33619
1	1	0.2	30	1	2	0.2	0.009	1.34207
1	1	0.5	30	1	2	0.2	0.009	1.36445
1	1	0.3	0	1	2	0.2	0.009	1.99932
1	1	0.3	20	1	2	0.2	0.009	1.70582
1	1	0.3	40	1	2	0.2	0.009	0.860745
1	1	0.3	30	0.3	2	0.2	0.009	1.5554
1	1	0.3	30	0.6	2	0.2	0.009	1.41782
1	1	0.3	30	2	2	0.2	0.009	1.33315
1	1	0.3	30	0.6	1	0.2	0.009	0.985001
1	1	0.3	30	0.6	3	0.2	0.009	1.90207
1	1	0.3	30	0.6	5	0.2	0.009	3.10153
1	1	0.3	30	0.6	7	0.2	0.009	4.93307
1	1	0.3	30	1	2	0.1	0.009	−0.30846
1	1	0.3	30	1	2	0.3	0.009	2.16293
1	1	0.3	30	1	2	0.4	0.009	2.59576
1	1	0.3	30	1	2	0.2	0.001	0.8657
1	1	0.3	30	1	2	0.2	0.015	2.17634
1	1	0.3	30	1	2	0.2	0.020	4.33893

Table 3. Nusselt number Nu and Sherwood number Sh for different values of parameters

B	B_1	Du	λ	Sc	Sr	t	Γ	Nu	Sh
1	1	0.05	30	1	2	0.2	0.009	0.990021	0.0551673
1	1	0.2	30	1	2	0.2	0.009	1.01465	0.00406177
1	1	0.5	30	1	2	0.2	0.009	1.14357	−0.243778
1	1	0.3	30	0.3	2	0.2	0.009	0.993081	0.152193
1	1	0.3	30	0.6	2	0.2	0.009	1.00958	0.0931558
1	1	0.3	30	2	2	0.2	0.009	1.20253	−0.848349
1	1	0.3	30	0.6	1	0.2	0.009	0.932046	0.531259
1	1	0.3	30	0.6	3	0.2	0.009	1.11071	−0.467361
1	1	0.3	30	0.6	5	0.2	0.009	2.46166	−7.52567
1	1	0.3	30	1	2	0.1	0.009	1.56902	0.13372
1	1	0.3	30	1	2	0.3	0.009	0.776413	−0.121012
1	1	0.3	30	1	2	0.4	0.009	0.60648	−0.159452

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