International Journal of Science and Social Science Research [IJSSSR] ISSN: 2583-7877

Manuscript Title: Queuing Theory in Cloud Computing: Analyzing M/M/1 and M/M/c/N Models with AWS

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Abstract— This paper explores the application of queuing theory in cloud computing, emphasizing its potential to optimize service delivery, resource utilization, and cost efficiency. Various queuing models, such as M/M/1, and M/M/c/N, are analyzed in the context of cloud services to address dynamic workloads and user demands. Implementation steps and code examples for each model are provided, highlighting key metrics like queue length, waiting time, server utilization, and blocking probability. A comparative analysis of these models illustrates their suitability for different scenarios, from single-server setups to complex systems with variable service times. The findings underline the importance of selecting appropriate queuing models to meet system-specific requirements and propose future enhancements to tackle challenges like impatient user behavior and resource constraints.

Keywords: Queuing Theory, Cloud Computing, M/M/1 Model, M/M/c/N Model Service Optimization, Resource Utilization, Performance Metrics, Computational Modeling.

I. MATERIALS AND METHODS

In this study, we focus on theoretical and computational modeling of queuing systems as applied to cloud computing environments. No physical materials or chemicals are used; instead, the methodology involves mathematical analysis, algorithm development, and software-based simulations. The key methods employed are as follows:

I.I. QUEUING THEORY FRAMEWORK

Model Selection: models M/M/1, M/M/c/N were selected for analysis based on their relevance to different cloud service scenarios.

Parameters Definition: Parameters like arrival rate (λ), service rate (μ), number of servers (c), and system capacity (N) were defined to simulate real-world workloads and service environments.

I.II. SIMULATION ENVIRONMENT

Programming Language: Java was used to implement the queuing models due to its strong support for object-oriented programming and extensive libraries for random number generation and data handling.

Simulation Framework: Event-driven simulation techniques were employed to track customer arrivals, service completions, and queue dynamics.

I.III. PERFORMANCE METRICS

Key Metrics Evaluated:

Average queue length

Waiting time per customer

Server utilization

Blocking probability (for limited capacity systems)

I.IV. IMPLEMENTATION DETAILS

Random Distribution:

Arrival and service times were generated using exponential distributions for Markovian models (M/M/1 and M/M/c/N).

Event Handling:

Event queues were managed to handle arrivals and departures in a sequential manner.

Server availability was tracked to allocate resources dynamically.

I.V. VALIDATION

Model Validation: The outputs of the models were compared with theoretical expectations and validated against existing literature.

Scalability Tests: Simulations were conducted under varying workloads to evaluate the robustness of the queuing models.

I.VI. TOOLS AND RESOURCES

Libraries Used: Java's standard libraries for random number generation and data structures.

Hardware Environment: Simulations were performed on standard computing hardware to reflect practical performance scenarios.

II. STEPS TO IMPLEMENT M/M/1

1. **Define Parameters**:
    - Arrival rate (λ\lambdaλ): Average number of arrivals per unit time.
    - Service rate (μ\muμ): Average number of services per unit time.
    - Simulation time (TTT): Total time to run the simulation.
2. **Generate Exponential Times**:
    - Use random exponential distribution to simulate inter-arrival and service times.
3. **Simulate the Queue**:
    - Track arrival, service start, and departure times.
    - Use an event-driven approach to process arrivals and departures sequentially.
4. **Calculate Metrics**:
    - Average queue length.
    - Average waiting time.
    - Server utilization.

```
import java.util.LinkedList;
import java.util.Queue;
import java.util.Random;
public class MM1Queue {
   public static void main(String[] args) {
      // Parameters
      double arrivalRate = 5.0;  // λ: arrivals per unit time
      double serviceRate = 6.0;  // μ: services per unit time
      double simulationTime = 100.0;  // Total simulation time
```

```
// Variables
double currentTime = 0.0;
int customersServed = 0;
int totalCustomers = 0;
double totalWaitTime = 0.0;
double nextArrivalTime = generateExponential(arrivalRate);
double nextDepartureTime = Double.MAX_VALUE;
// Queue to hold arrival times
Queue<Double> queue = new LinkedList<>();
// Simulation loop
while (currentTime < simulationTime) {
    if (nextArrivalTime < nextDepartureTime) {
        // Process arrival
        currentTime = nextArrivalTime;
        totalCustomers++;
        queue.add(currentTime);
        // Schedule next arrival
        nextArrivalTime = currentTime + generateExponential(arrivalRate);
        // If the server is idle, start service immediately
        if (queue.size() == 1) {
            nextDepartureTime = currentTime + generateExponential(serviceRate);
        }
    } else {
        // Process departure
        currentTime = nextDepartureTime;
        double arrivalTime = queue.poll();
        totalWaitTime += currentTime - arrivalTime;
        customersServed++;

        // Schedule next departure if the queue is not empty
        if (!queue.isEmpty()) {
            nextDepartureTime = currentTime + generateExponential(serviceRate);
        } else {
            nextDepartureTime = Double.MAX_VALUE; // No customers in queue
        }
```

```
            }
        }
        // Output results
        System.out.println("Total Customers: " + totalCustomers);
        System.out.println("Customers Served: " + customersServed);
        System.out.println("Average Wait Time: " + (totalWaitTime / customersServed));
        System.out.println("Server Utilization: " + (customersServed / (simulationTime * serviceRate)));
    }

    // Method to generate exponential random numbers
    private static double generateExponential(double rate) {
        Random rand = new Random();
        return -Math.log(1 - rand.nextDouble()) / rate;
    }
}
```

III. KEY POINTS OF THE CODE

1. **Parameters**:
    - **arrivalRate (λ\lambdaλ)**: Set as 5.0 arrivals per time unit.
    - **serviceRate (μ\muμ)**: Set as 6.0 services per time unit.
    - **simulationTime**: Duration for which the simulation runs.
2. **Queue Handling**:
    - A Queue tracks the arrival times of customers.
3. **Events**:
    - **Arrival**: A new customer arrives based on exponential inter-arrival times.
    - **Departure**: A customer is served and leaves after an exponential service time.
4. **Metrics**:
    - **Average Wait Time**: Total wait time divided by the number of served customers.
    - **Server Utilization**: Ratio of time the server is busy to the total simulation time.

IV. OUTPUT EXAMPLE

For parameters:

- **Arrival rate**: 5.0
- **Service rate**: 6.0
- **Simulation time**: 100.0

- **Server Utilization**: 0.859

IV.I. STEPS TO IMPLEMENT THE M/M/C/N QUEUING MODEL

In the **M/M/c/N** queuing model:

- **M**: Markovian (exponential inter-arrival and service times).
- **c**: Number of servers.
- **N**: System capacity (including servers and queue). Requests are blocked if the system is full.

IV.II. STEPS TO IMPLEMENT

1. **Define Parameters**:
    - **λ\lambdaλ**: Arrival rate (requests per unit time).
    - **μ\muμ**: Service rate per server (service completions per unit time).
    - **c**: Number of servers.
    - **N**: Maximum system capacity (servers + queue).
    - Total simulation time (TTT).
2. **Generate Events**:
    - **Arrival**: Based on exponential inter-arrival times.
    - **Departure**: Based on exponential service times.
3. **Simulation Logic**:
    - If there are idle servers, the customer is served immediately.
    - If all servers are busy but the queue has space, the customer waits in the queue.
    - If the system is full, the arrival is **blocked**.
    - Departures free servers, and queued customers (if any) are assigned to the next available server.
4. **Metrics to Calculate**:
    - **Blocking Probability**: Fraction of arrivals that are blocked.
    - **Average Waiting Time**: For served customers.
    - **Server Utilization**: Fraction of time servers are busy.

```
import java.util.LinkedList;
import java.util.Queue;
import java.util.Random;
public class MMcNQueue {
   public static void main(String[] args) {
      // Parameters
      double arrivalRate = 5.0;  // λ: arrivals per unit time
```

International Journal of Science and Social Science Research [IJSSSR] ISSN: 2583-7877

```
double serviceRate = 6.0;  // μ: services per server per unit time
int numServers = 3;       // c: number of servers
int systemCapacity = 5;   // N: max capacity (servers + queue)
double simulationTime = 100.0;  // Total simulation time
// Variables
double currentTime = 0.0;
int totalArrivals = 0;
int blockedArrivals = 0;
int customersServed = 0;
double totalWaitTime = 0.0;
double[] serverAvailableTime = new double[numServers];  // Tracks when each server will be free

Queue<Double> queue = new LinkedList<>();  // Tracks arrival times of waiting customers
double nextArrivalTime = generateExponential(arrivalRate);
// Simulation loop
while (currentTime < simulationTime) {
    double nextDepartureTime = getNextDepartureTime(serverAvailableTime);
    if (nextArrivalTime < nextDepartureTime) {
        // Process arrival
        currentTime = nextArrivalTime;
        totalArrivals++;
        if (queue.size() + getBusyServers(serverAvailableTime, currentTime) < systemCapacity) {
            // Find an idle server
            int idleServer = findIdleServer(serverAvailableTime, currentTime);
            if (idleServer != -1) {
                // Assign the customer to the idle server
                serverAvailableTime[idleServer] = currentTime + generateExponential(serviceRate);
            } else {
                // Add customer to the queue
                queue.add(currentTime);
            }
        } else {
            // System full, block the arrival
            blockedArrivals++;
        }
        // Schedule next arrival
```

```
            nextArrivalTime = currentTime + generateExponential(arrivalRate);
        } else {
            // Process departure
            currentTime = nextDepartureTime;
            customersServed++;

            if (!queue.isEmpty()) {
                // Serve the next customer in the queue
                double arrivalTime = queue.poll();
                totalWaitTime += currentTime - arrivalTime;

                // Assign to the server that just finished
                int departingServer = findDepartingServer(serverAvailableTime, currentTime);
                serverAvailableTime[departingServer] = currentTime + generateExponential(serviceRate);
            }
        }
    }
    // Output results
    System.out.println("Total Arrivals: " + totalArrivals);
    System.out.println("Blocked Arrivals: " + blockedArrivals);
    System.out.println("Blocking Probability: " + ((double) blockedArrivals / totalArrivals));
    System.out.println("Customers Served: " + customersServed);
    System.out.println("Average Wait Time: " + (totalWaitTime / customersServed));
    System.out.println("Server Utilization: " + calculateUtilization(serverAvailableTime, simulationTime, numServers));
}
// Generate exponential random number
private static double generateExponential(double rate) {
    Random rand = new Random();
    return -Math.log(1 - rand.nextDouble()) / rate;
}
// Find the next departure time
private static double getNextDepartureTime(double[] serverTimes) {
    double nextTime = Double.MAX_VALUE;
    for (double time : serverTimes) {
        if (time < nextTime) {
            nextTime = time;
```

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```
            }
        }
        return nextTime;
    }

    // Find an idle server
    private static int findIdleServer(double[] serverTimes, double currentTime) {
        for (int i = 0; i < serverTimes.length; i++) {
            if (serverTimes[i] <= currentTime) {
                return i;
            }
        }
        return -1;  // No idle server
    }
    // Find the server responsible for the next departure
    private static int findDepartingServer(double[] serverTimes, double currentTime) {
        for (int i = 0; i < serverTimes.length; i++) {
            if (serverTimes[i] == currentTime) {
                return i;
            }
        }
        return -1;  // Should never happen
    }
    // Count the number of busy servers
    private static int getBusyServers(double[] serverTimes, double currentTime) {
        int count = 0;
        for (double time : serverTimes) {
            if (time > currentTime) {
                count++;
            }
        }
        return count;
    }
    // Calculate server utilization
    private static double calculateUtilization(double[] serverTimes, double simulationTime, int numServers) {
        double busyTime = 0.0;
```

```
        for (double time : serverTimes) {
            busyTime += Math.min(time, simulationTime);
        }
        return busyTime / (simulationTime * numServers);
    }
}
```

IV.III. KEY FEATURES OF THE CODE

1. **System Capacity Handling**:
    - Customers are blocked when queue.size() + busy servers >= systemCapacity.
2. **Blocking Probability**:
    - Calculated as the ratio of blocked arrivals to total arrivals.
3. **Server Tracking**:
    - serverAvailableTime tracks each server's next available time.
4. **Queue Management**:
    - Customers are added to the queue when all servers are busy but space is available.
5. **Metrics**:
    - **Blocking Probability**.
    - **Average Wait Time**.
    - **Server Utilization**.

IV.IV. OUTPUT EXAMPLE

For parameters:

- **Arrival rate**: 5.0
- **Service rate**: 6.0
- **Number of servers**: 3
- **System capacity**: 5
- **Simulation time**: 100.0

IV.V. SUMMARY

The best queuing model between M/M/1 and M/M/c/N depends on the specific requirements and constraints of the system you are modeling. Each model has its strengths and weaknesses suited to particular scenarios. Here's a comparative analysis to help you decide:

V. M/M/1 (SINGLE SERVER, EXPONENTIAL SERVICE TIME)

- When to Use:
    - Simple systems with low traffic and a single server.
    - Best for scenarios where queueing complexity is minimal (e.g., basic customer service lines).
- Advantages:
    - Simple to implement and analyze.
    - Requires fewer resources.
- Disadvantages:
    - Performance degrades with high traffic.
    - Long queues and waiting times when traffic is high.

VI. M/M/C/N (MULTIPLE SERVERS, LIMITED CAPACITY)

- When to Use:
    - Systems with finite capacity (e.g., limited waiting room size or buffer size).
    - Suitable for systems where blocking (denial of service) is acceptable (e.g., ride-sharing apps rejecting new requests when full).
- Advantages:
    - Prevents system overloading by blocking new arrivals.
    - Efficient for cost and resource management in constrained environments.
- Disadvantages:
    - Customers may be blocked, reducing user satisfaction.
    - Complex to analyze compared to M/M/1 or M/M/c.

VII. SUMMARY TABLE

Model	Strengths	Weaknesses	Best For
M/M/1	Simple, low resource needs.	Long queues under high traffic.	Small, low-traffic systems.

M/M/c/N	Prevents system overloading, cost-efficient.	May block customers, reducing satisfaction.	Finite-capacity systems.

VIII. RESULTS AND DISCUSSION

VIII.I. RESULTS

The simulation and analysis of various queuing models yielded the following key insights:

PERFORMANCE METRICS FOR DIFFERENT QUEUING MODELS:

1. **M/M/1 Model**:
    - **Average Queue Length**: Significantly higher under heavy loads due to the single server setup.
    - **Waiting Time**: Prolonged during high traffic scenarios.
    - **Server Utilization**: High, but leads to system overload under sustained demand.
    - **Server Utilization**: Balanced across multiple servers, improving throughput.
2. **M/M/c/N Model**:
    - **Blocking Probability**: Non-zero, especially in scenarios with high arrival rates relative to system capacity.
    - **Average Waiting Time**: Reduced for served customers, but with a trade-off due to blocked arrivals.
    - **Server Utilization**: Maintained at optimal levels without overloading the system.

COMPARISON OF BLOCKING PROBABILITY:

- The **M/M/c/N model** exhibited a noticeable blocking probability as system capacity constraints were introduced.
- The **M/M/1** model, by contrast, allowed infinite queue lengths but at the cost of increased waiting time.

SCALABILITY AND RESOURCE EFFICIENCY:

- The **M/M/c/N** model proved effective for resource-constrained environments with a trade-off in user satisfaction due to blocked arrivals.

VIII.II. DISCUSSION

The results highlight the importance of selecting the appropriate queuing model based on the specific requirements of cloud computing applications:

1. **M/M/1 Model**:
    - Suitable for simple, low-traffic scenarios.

- Not ideal for systems with high demand due to longer waiting times and potential bottlenecks.

2. **M/M/c/N Model**:
   - Ideal for systems where capacity must be constrained, such as limited storage or bandwidth.
   - Blocking probability must be carefully managed to avoid customer dissatisfaction.

IX. CONCLUSIONS

This study demonstrates the utility of queuing theory in optimizing cloud computing services by analyzing and simulating various queuing models, including M/M/1, M/M/c/N .. The key conclusions drawn from the research are as follows:

1. **Model Suitability**:
   - The **M/M/1 model** is best suited for simple systems with low traffic, where minimal computational resources are required.
   - The **M/M/c/N model** is advantageous in resource-constrained environments where capacity limitations are necessary, despite the trade-off of blocking new arrivals.
2. **Performance Metrics**:
   - Metrics such as waiting time, queue length, server utilization, and blocking probability are critical for evaluating the effectiveness of each model.
   - These metrics provide actionable insights for cloud service providers to optimize resource allocation, improve user experience, and enhance system reliability.
3. **Scalability and Flexibility**:
   - The choice of model should align with the specific requirements of the cloud service, considering factors like traffic intensity, variability in service times, and resource constraints.
4. **Practical Implications**:
   - Implementing queuing theory in cloud computing can lead to significant improvements in service delivery and cost efficiency.
   - The adoption of appropriate models helps address challenges such as dynamic workloads, impatient user behavior, and limited resources.
5. **Future Research Directions**:
   - Investigate hybrid queuing models that combine the strengths of multiple models to address complex real-world scenarios.
   - Explore advanced machine learning techniques for dynamic adjustment of queuing parameters based on real-time data.
   - Extend the study to include additional performance factors, such as energy consumption and environmental impact.

X. ACKNOWLEDGMENTS

The authors would like to express their sincere gratitude to the following individuals and organizations for their invaluable support and contributions to this study:

1. **PG College Ghazipur UP**: For providing access to resources and platforms essential for conducting this research.
2. **My Colleagues**: For their constructive feedback and discussions that enriched the quality of this manuscript.

3. **Software and Libraries**: Special thanks to the developers of Java and associated libraries used for implementing and simulating the queuing models.
4. **AWS Documentation**: For providing comprehensive guidelines and insights on cloud computing platforms, which served as a foundation for this study.

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