On Fully Fuzzy Linear Programming Problems

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Abstract—The fuzzy linear programming problem has been used as an important tool in different disciplines such as engineering, business, economics, commerce, defence etc. Fully Fuzzy Linear Programming Problems (FFLP) are those in which all the parameters that is the cost coefficients, the technological coefficients, the right-hand side of the constraints, and the decision variable are fuzzy numbers. Ezzati et al. [1] suggested an algorithm to solve fully fuzzy linear programming problems. In this paper, we propose two methods to find the optimal solution of fully fuzzy linear programming problems by the defuzzification of the parameters in the objective function with fuzzy inequality constraints. We consider here the cases of triangular fuzzy numbers.

Keywords: Fully fuzzy linear programming problems, triangular fuzzy number, fuzzy solution.

I. INTRODUCTION

Linear Programming Problems (LPP) is one of the most powerful and useful techniques for getting solutions of decision-making problems. Linear programming problems can be used for solving problems arising in engineering, business, economics, commerce, defence etc. In conventional linear programming problems, the data have crisp values. But in many real-life problems, the data are usually imprecise that is the data are not crisp. In order to deal with such data LA Zadeh [4] introduced fuzzy sets and eventually fuzzy numbers in 1965. Fuzzy Linear Problems (FLPP) are those LPPs where either the cost coefficients or the technological coefficients or the right-hand sides of the constraints are fuzzy numbers. Fuzzy Linear Programming Problems (FLPP) allow working with imprecise data and constraints leading to more realistic models. They have often been used for solving a wide variety of problems in sciences and engineering. A number of researchers have been working on fuzzy linear programming problems. Tanaka et al. [2] was one of the first researchers working on fuzzy linear programming problems based on the fuzzy decision framework of Bellman and Zadeh [3]. Since then, numerous researchers have studied various properties of FLP problems and proposed different approaches for solving them. Fully Fuzzy Linear Programming Problems (FFLPP) are those in which all the parameters that is the cost coefficients, the technological coefficients, the right-hand side of the constraints, and the decision variable are fuzzy numbers. Ezzati et al. [1] suggested an algorithm to solve fully fuzzy linear programming problems. In this paper, we propose two methods to find the fuzzy optimal solution of fully fuzzy linear programming problems by defuzzification of the parameters in the objective function with fuzzy inequality constraints. We consider here the case of triangular fuzzy numbers. In section II we give some terminologies in fuzzy sets and in section III, results on FFLPP are discussed in two different methods with an example.

II. PRELIMINARIES

Definition: 1 (Fuzzy Set): Let X be a crisp universal set. Then a fuzzy set $\tilde{A}$ in X is defined by its membership function $\mu_{\tilde{A}}: X \to [0,1]$ which assigns a real number denoted by $\mu_{\tilde{A}}(x)$ in the interval [0,1]; to each element x of X, where $\mu_{\tilde{A}}(x)$ represents the grade of membership of x in $\tilde{A}$.

Definition: 2 (Support of a fuzzy set): The support of a fuzzy set $\tilde{A}$ in X is the crisp set of those points of X at which $\mu_{\tilde{A}}(x)$ is positive and is denoted by $Supp(\tilde{A})$. That is $Supp(\tilde{A}) = \{x \in X: \mu_{\tilde{A}}(x) > 0\} \subset X$.

Definition: 3 (Height of a fuzzy set): The height of a fuzzy set $\tilde{A}$ is denoted and defined by $Hgt(\tilde{A}) = Sup\{\mu_{\tilde{A}}(x): x \in X\}$.

Definition: 4 (Normal fuzzy set): A fuzzy set $\tilde{A}$ in X is called a normal fuzzy set if its height is unity, i.e., $\mu_{\tilde{A}}(x) = 1$ for some $x \in X$. If a fuzzy set is not normal it is called a subnormal fuzzy set.

Definition: 5 (Convex fuzzy set): A fuzzy set $\tilde{A}$ in $\mathbb{R}$ is said to be convex if $\forall x_1, x_2 \in \mathbb{R}$, and $\forall \lambda \in [0,1]$, $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$.

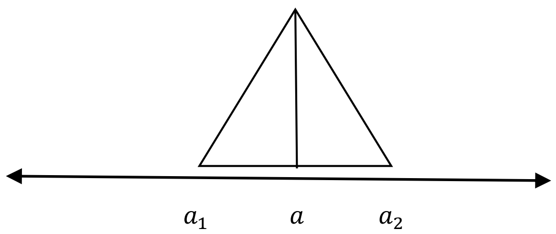
Definition: 6 (Fuzzy number): A fuzzy number is a fuzzy set on the real line $\mathbb{R}$ which is normal, convex and its membership function is piecewise continuous.

The class of all fuzzy numbers is denoted by $F(\mathbb{R})$.

A fuzzy number $\tilde{A}$ is called a positive fuzzy number if $\forall x < 0$, $\mu_{\tilde{A}}(x) = 0$ and a negative fuzzy number if $\forall x > 0$, $\mu_{\tilde{A}}(x) = 0$.

Definition: 7 (Triangular fuzzy number): A fuzzy number $\tilde{a}$ denoted by $\tilde{a} = (a_1, a, a_2)$ is called a triangular fuzzy number if its membership function $\mu_{\tilde{a}}(x)$ is given by

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, x < a_1 \\ \dfrac{x - a_1}{a - a_1}, a_1 \leq x \leq a \\ \dfrac{a_2 - x}{a_{2-a}}, a \leq x \leq a_2 \\ 0, x > a_2 \end{cases}$$



Operations on fuzzy numbers defined:

We define the addition, subtraction, multiplication between two triangular fuzzy numbers and the scalar multiplication as follows:

For $\tilde{a} = (a_1, a, a_2)$, $\tilde{b} = (b_1, b, b_2)$ and $k \in \mathbb{R}$, where $a_1 \geq 0, b_1 \geq 0$

$$\tilde{a} + \tilde{b} = (a_1 + b_1, a + b, a_2 + b_2)$$

$$\tilde{a} - \tilde{b} = (a_1 - b_2, a - b, a_2 - b_1)$$

$$\tilde{a}\tilde{b} = (a_1 b_1, ab, a_2 b_2)$$

$$k\tilde{a} = \begin{cases} (ka_1, ka, ka_2), k \geq 0 \\ (ka_2, ka, 4a_1), k < 0 \end{cases}$$

Ranking and defuzzification of triangular fuzzy numbers:

If $\tilde{a} = (a_1, a, a_2)$, we define the ranking of $\tilde{a}$ by $R(\tilde{a}) = \frac{a_1 + 2a + a_2}{4}$.

Degree of fuzziness: For $\tilde{a} = (a_1, a, a_2)$, we define the degree of fuzziness of $\tilde{a}$ as follows: $DF(\tilde{a}) = a_2 - a_1$.

Proposition: Let $\tilde{a}$ be a triangular fuzzy number with degree of fuzziness D and $R(\tilde{a}) = R$. Then $\tilde{a}$ can be written in the form $\tilde{a} = (a, 3R - 2a - D, a + D)$, where $a \in \mathbb{R}$ such that $R - \frac{2D}{3} < a < R - \frac{D}{3}$.

Comparison of two triangular fuzzy numbers:

For $\tilde{a} = (a_1, a, a_2)$ and $\tilde{b} = (b_1, b, b_2)$ we define $\tilde{a} = \tilde{b} \Leftrightarrow a_1 = b_1, a = b, a_2 = b_2$ and $\tilde{a} \leq \tilde{b} \Leftrightarrow b - b_1 \leq a - a_1, a \leq b, a_2 - a \leq b_2 - b$

III. METHOD OF SOLVING FULLY FUZZY LINEAR PROGRAMMING PROBLEM (FFLPP)

We consider the Fully Fuzzy Linear Programming Problem (FLPP)

$max\, \tilde{Z} = \tilde{c}^T\tilde{x}$, subject to the constraints $\tilde{A}\tilde{x} \leq \tilde{b}, \tilde{x}$ is positive, where $\tilde{c} = (\tilde{c}_1, \tilde{c}_2 \cdots\cdots\cdots\cdots \tilde{c}_n)^T$, $\tilde{x} = \left(\tilde{x}_1, \tilde{x}_{2,\dots,}\tilde{x}_n\right)^T$, $\tilde{A} = \left(\tilde{a}_{ij}\right)_{m\times n}$, and $\tilde{b} = \left(\tilde{b}_1, \tilde{b}_2, \dots, \tilde{b}_m\right)^T$. That is

$max\, \tilde{Z} = \tilde{c}_1\tilde{x}_1 + \tilde{c}_2\tilde{x}_2 + \cdots + \tilde{c}_n\tilde{x}_n$, subject to the constraints

$\tilde{a}_{11}\tilde{x}_1 + \tilde{a}_{12}\tilde{x}_2 + \cdots + \tilde{a}_{1n}\tilde{x}_n \leq \tilde{b}_1$, $\tilde{a}_{21}\tilde{x}_1 + \tilde{a}_{22}\tilde{x}_2 + \cdots + \tilde{a}_{2n}\tilde{x}_n \leq \tilde{b}_2,\dots$, $\tilde{a}_{m1}\tilde{x}_1 + \tilde{a}_{m2}\tilde{x}_2 + \cdots + \tilde{a}_{mn}\tilde{x}_n \leq \tilde{b}_m$.

Let $\tilde{c}_j = \left(c_j', c_j, c_j''\right)$, $\tilde{x}_j = \left(x_j', x_j, x_j''\right)$, $\tilde{a}_{ij} = \left(a_{ij}', a_{ij}, a_{ij}''\right)$, $\tilde{b}_i = (b_i', b_i, b_i'')$, and $x_j' \geq 0$ for each $i = 1,2, \dots, m$ and $j = 1,2, \dots, n$.

Case 1: We defuzzify each of these fuzzy parameters. The problem is then reduced to a crisp linear programming problem and we get the fuzzy solution $\tilde{x}_j$ of the given problem.

Case 2: We apply the operations of multiplication, scalar multiplication, addition, and subtraction between two fuzzy numbers defined earlier and finally applying the inequality between two triangular fuzzy numbers, we convert the given FLPP into a crisp LPP and then we solve it by the simplex method and finally get the fuzzy solution $\tilde{x}_j$ of the given problem.

The process is explained with the help of following examples.

Examples: We consider an example

$max\, \tilde{Z} = (4,5,6)(x_1', x_1, x_1'') + (2,3,4)(x_2', x_2, x_2'')$, subject to the constraints

$$(2,3,4)(x_1', x_1, x_1'') + (4,5,6)(x_2', x_2, x_2'') \leq (14,15,16)$$

$$(4,5,6)(x_1', x_1, x_1'') + (1,2,3)(x_2', x_2, x_2'') \leq (9,10,11)$$

$$x_1' \geq 0, x_2' \geq 0$$

The objective function is made a crisp objective function first by defuzzifying the parameters and then secondly by multiplying and then defuzzifying subject to the given set of constraints. Thus, forming the two crisp LPPs as follows:

1. $max\, Z = \frac{5}{4}(x_1' + 2x_1 + x_1'') + \frac{3}{4}(x_2' + 2x_2 + x_2'')$, subject to the constraints

$$(2x_1', 3x_1, 4x_1'') + (4x_2', 5x_2, 6x_2'') \leq (14,15,16)$$

$$(4x_1', 5x_1, 6x_1'') + (x_2', 2x_2, 3x_2'') \leq (9,10,11)$$

$$x_1', x_1, x_1'', x_2', x_2, x_2'' \geq 0$$

That is $max\, Z = 1.25x_1' + 2.5x_1 + 1.25x_1'' + 0.75x_2' + 1.5x_2 + 0.75x_2''$, subject to the constraints

$$(2x_1' + 4x_2', 3x_1 + 5x_2, 4x_1'' + 6x_2'') \leq (14,15,16)$$

$$(4x_1' + x_2', 5x_1 + 2x_2, 6x_1'' + 3x_2'') \leq (9,10,11)$$

$$x_1', x_1, x_1'', x_2', x_2, x_2'' \geq 0$$

That is $max\, Z = 1.25x_1' + 2.5x_1 + 1.25x_1'' + 0.75x_2' + 1.5x_2 + 0.75x_2''$, subject to the constraints

$$-2x_1' + 3x_1 + 0x_1'' - 4x_2' + 5x_2 + 0x_2'' \geq 1$$

$$0x_1' + 3x_1 + 0x_1'' + 0x_2' + 5x_2 + 0x_2'' \leq 15$$

$$0x_1' - 3x_1 + 4x_1'' + 0x_2' - 5x_2 + 6x_2'' \leq 1$$

$$-4x_1' + 5x_1 + 0x_1'' - x_2' + 2x_2 + 0x_2'' \geq 1$$

$$0x_1' + 5x_1 + 0x_1'' + 0x_2' + 2x_2 + 0x_2'' \leq 10$$

$$0x_1' - 5x_1 + 6x_1'' + 0x_2' - 2x_2 + 3x_2'' \leq 1$$

$$x_1', x_1, x_1'', x_2', x_2, x_2'' \geq 0$$

2. $\max \tilde{Z} = (4x', 5x_1, 6x_1'') + (2x_2', 3x_2, 4x_2'')$, subject to the constraints

$$(2x_1', 3x_1, 4x_1'') + (4x_2', 5x_2, 6x_2'') \leq (14{,}15{,}16)$$

$$(4x_1', 5x_1, 6x_1'') + (x_2', 2x_2, 3x_2'') \leq (9{,}10{,}11)$$

$$x_1', x_1, x_1'', x_2', x_2, x_2'' \geq 0$$

That is $max\, \tilde{Z} = (4x_1' + 2x_2', 5x_1 + 3x_2, 6x_1'' + 4x_2'')$, subject to the constraints

$$(2x_1' + 4x_2', 3x_1 + 5x_2, 4x_1'' + 6x_2'') \leq (14{,}15{,}16)$$

$$(4x_1' + x_2', 5x_1 + 2x_2, 6x_1'' + 3x_2'') \leq (9{,}10{,}11)$$

$$x_1', x_1, x_1'', x_2', x_2, x_2'' \geq 0$$

That is $max\, Z = x_1' + 2.5x_1 + 1.5x_1'' + 0.5x_2' + 1.5x_2 + x_2''$, subject to the constraints

$$-2x_1' + 3x_1 + 0x_1'' - 4x_2' + 5x_2 + 0x_2'' \geq 1$$

$$0x_1' + 3x_1 + 0x_1'' + 0x_2' + 5x_2 + 0x_2'' \leq 15$$

$$0x_1' - 3x_1 + 4x_1'' + 0x_2' - 5x_2 + 6x_2'' \leq 1$$

$$-4x_1' + 5x_1 + 0x_1'' - x_2' + 2x_2 + 0x_2'' \geq 1$$

$$0x_1' + 5x_1 + 0x_1'' + 0x_2' + 2x_2 + 0x_2'' \leq 10$$

$$0x_1' - 5x_1 + 6x_1'' + 0x_2' - 2x_2 + 3x_2'' \leq 1$$

$$x_1', x_1, x_1'', x_2', x_2, x_2'' \geq 0$$

The solutions of these two LPPs are as follows:

1. $R(\tilde{x}_1) = 1.1066, R(\tilde{x}_2) = 2.4044, max\, \tilde{Z} = 12.7467$

2. $R(\tilde{x}_1) = 1.1065, R(\tilde{x}_2) = 2.4044, max\ \tilde{Z} = 12.4044$

These solutions can be fuzzified with the help of the proposition mentioned above if need be.

IV. CONCLUSION

This work develops two techniques to solve Fully Fuzzy Linear Programming problems by defining fuzzy arithmetic operations and fuzzy inequality between two triangular fuzzy numbers. The methods are then explained with an example. Then we compare the result obtained by the two different methods and it is found that the first method gives better result. An analogous method could be applied for solving FFLPP with trapezoidal fuzzy numbers.

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