

Impact of carbon emission and learning effect on dual channel supply chain with deteriorating items

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Abstract— This study examines inventory and pricing decisions in a dual-channel supply chain for deteriorating products, where demand is influenced by price, advertisement, and time dependency, along with the learning effect and inflationary factors. The supply chain consists of both online and offline sales channels, where the deterioration of products over time significantly affects decision-making. A mathematical model is developed to capture the dynamic interactions between pricing strategies, advertising efforts, and inventory control, considering demand fluctuations over time. Additionally, the model incorporates the learning effect, which reflects improvements in production and cost efficiency, as well as the impact of inflation on product pricing and costs. The analysis provides insights into how supply chain managers can optimize inventory levels and pricing policies to balance profitability and customer demand in a rapidly changing environment. Key factors such as product deterioration, inflation, and evolving consumer behaviour are examined to formulate effective strategies for managing dual-channel operations. Numerical experiments are conducted to validate the model and highlight the practical implications of the findings for supply chain optimization.

Keywords- dual channel supply chain, inflation, deterioration, advertisement dependent demand

I. INTRODUCTION

In today’s highly competitive and rapidly evolving market environment, dual-channel supply chains, consisting of both online and offline sales platforms, have become an essential strategy for companies aiming to expand their market reach and enhance customer satisfaction. These supply chains must address numerous challenges related to demand management, pricing strategies, and inventory control, especially when dealing with deteriorating products. Products such as food, pharmaceuticals, and high-tech goods often experience quality degradation over time, making it crucial for companies to carefully manage inventory levels while optimizing pricing decisions to mitigate the losses due to product deterioration.

In addition to product deterioration, several other factors significantly influence the demand for products. Demand is highly sensitive to pricing strategies and advertising efforts, where both price and advertisement investments play a crucial role in shaping consumer preferences. Moreover, demand often evolves over time, influenced by seasonal effects, market trends, and consumer learning. The learning effect refers to the phenomenon where consumers or producers become more efficient or knowledgeable over time, reducing costs or altering demand dynamics.

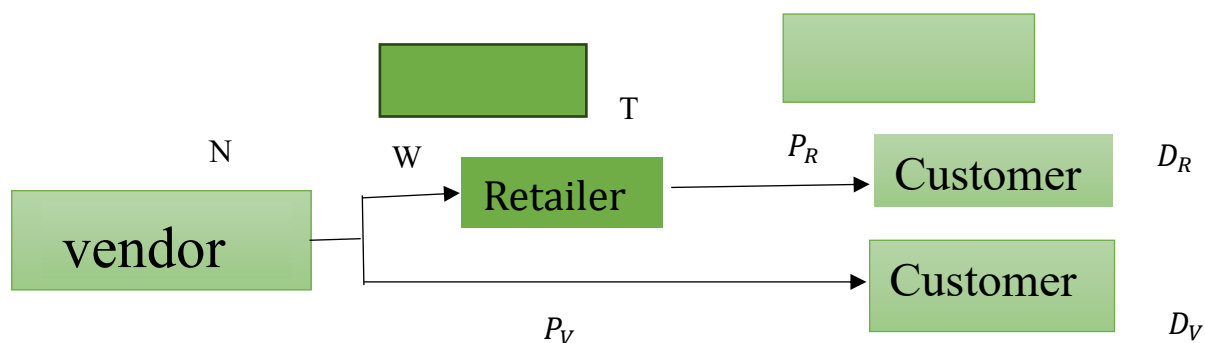


Fig:1- Dual Channel supply chain

Further complicating decision-making is the impact of inflation, which affects both production costs and consumer purchasing power. Inflationary pressures make it challenging for supply chain managers to maintain optimal pricing strategies while keeping inventory levels cost-effective. As inflation rises, companies must balance the increase in production costs with consumer price sensitivity, especially in a dual-channel system where price discrepancies between online and offline channels can influence

customer behaviour. This study aims to develop an integrated model that addresses the complex interplay of these factors—deterioration of products, price and advertisement-dependent demand, the learning effect, and inflation—in a dual-channel supply chain environment. By considering these dynamics holistically, the study seeks to provide valuable insights into optimizing inventory and pricing decisions to improve overall supply chain performance. Specifically, it explores how supply chain managers can adjust pricing, advertising efforts, and inventory levels in response to time-dependent demand and inflationary conditions while accounting for product deterioration.

The rest of the paper is organized as follows: The literature review discusses relevant studies on dual-channel supply chains, deteriorating products, and demand dependency. The model development section introduces the mathematical model used to analyse the decision variables, followed by solution approaches and a discussion of numerical results. Finally, conclusions are drawn, and managerial insights are offered for effective supply chain management in dual-channel systems with deteriorating products.

II. LITERATURE REVIEW

Carbon emissions have emerged as a critical factor in supply chain management due to increasing regulatory and societal pressures for sustainability. Studies have highlighted the integration of carbon emission considerations into supply chain. Dual-channel supply chains, which integrate traditional retail and online channels, have gained prominence due to evolving consumer behaviour and technological advancements. Chiang *et al.* (2003) argued that one of the key motivations for introducing a direct channel is to mitigate the effects of double marginalization. Similarly, Yan and Pei (2009) highlighted that establishing a direct channel serves as an effective strategy for manufacturers to incentivize retailers to enhance their service levels and improve the overall efficiency of the supply chain. Beyond pricing decisions, researchers have explored other critical and practical factors, including delivery lead time for direct channels, service levels across both channels, demand disruptions, and asymmetric information. Hua *et al.* (2010) demonstrated that in a dual-channel supply chain, lead time significantly influences pricing and quantity decisions for both parties. Extending the work of Chiang *et al.* (2003), Xu *et al.* (2012) treated the online channel's delivery time as a decision variable, emphasizing its strategic importance. Hammami *et al.* (2015) created a deterministic optimization model that accounts for carbon emissions in a multi-echelon production-inventory model with lead time limitations. They require that each client order be delivered within the due date specified by the customer. The quantity that cannot be provided on time results in a lost sale. They analyse a supply chain with many tiers, including external suppliers, production sites, and distribution locations. Darom *et al.* (2018) proposed a recovery model for a two-stage serial supply chain that has experienced supply disruption, taking into account safety stock and carbon emissions. The system is made up of a single manufacturer and a single retailer, with the carbon emissions coming from transportation operations during the recovery cycle. The model can determine the manufacturer's and retailer's new recovery schedules, the best safety stock level, and the influence of carbon emissions on recovery costs. He *et al.* (2020) investigated a single-retailer-single-vendor dual-channel supply chain model in which the vendor offers degrading items via both the direct internet and indirect retail channels. In addition to quantity depreciation, product quality deteriorates with time, affecting retail demand. The price and inventory choices for the two businesses are examined concurrently. Le and Mizuno (2022) investigated a periodic review, joint dynamic pricing, and inventory problem in a dual-channel supply chain with a single producer and retailer, where demand is stochastic and price sensitive. There are three different power arrangements between manufacturer and retailer in the dual-channel supply chain: Manufacturer Stackelberg, Retailer Stackelberg, and Vertical Nash. Das *et al.* (2023) in this research they take into account carbon emissions that occur throughout the manufacture and storage of products in the supplier inventory system, with the retailer accountable for absorbing the expenses associated with each emission created during product transit and storage. The unit production cost is determined by the production rate, and shortages of goods are permitted if they are substantially backordered. The management of deteriorating items, such as perishable goods or products with finite shelf lives, poses unique challenges. Advertisement plays a pivotal role in shaping consumer demand, particularly in dual-channel systems. Liu *et al.* (2024) introduced a novel ambiguity distribution set to represent uncertain demand. Based on this proposed distribution set, a robust fuzzy bi-level optimization pricing model is developed for the low-carbon direct channel (DC) supply chain. To address the manufacturer's compliance with carbon allowance trading (CAT) regulations, three regulatory mechanisms are considered: no CAT regulation, the grandfathering (GF) mechanism, and the benchmarking (BM) mechanism.

The integration of carbon emissions, learning effects, and inflationary factors into dual-channel supply chains dealing with deteriorating items and advertisement-driven demand remains a fertile area for research. Future studies could focus on developing comprehensive models that simultaneously address these interrelated factors, offering practical insights for sustainable and cost-effective supply chain.

III. NOTATIONS

In this model we use following notations throughout the study-

Symbol	Description
N	Multiple of retailer's cycle time
W	Whole sale price of the vendor
T	Total cycle time (retailer's replenishment)
P_R	Selling price for retailer channel
P_V	Selling price for vendor channel
D_R	Demand rate for retailer
D_V	Demand rate for vendor
Q_V	Ordering quantity for vendor
Q_{Ri}	Ordering quantity for retailer in phase i
A	Frequency of advertisement
α	Constant $0 < \alpha < 1$
β	Degree of product substitution of two channel
σ	Quality dropping rate
r	Rate of inflation
θ	Deterioration rate for retailer and vendor
C_{hR}	Holding cost for retailer
C_{dR}	Deterioration cost for retailer
O_R	Ordering cost for retailer
O'_R	Ordering cost for retailer due to carbon emission
C'_{hR}	Holding cost for retailer due to carbon emission
W'	Whole sale price of the vendor due to carbon emission
C'_{dR}	Deterioration cost for retailer due to carbon emission
O_V	Ordering cost for vendor
O'_V	Ordering cost for vendor due to carbon emission
C'_{hV}	Holding cost for vendor
C'_V	Purchasing cost for vendor
C_{hV}	Holding cost for vendor
O''_R	Constant
O''_V	Constant
C''_{hR}	Constant
C''_{hV}	Constant
ω	Learning rate
n	Cumulative frequency of shipment

IV. ASSUMPTIONS

The following assumption are considered for this inventory model:

1. In this model delivery time is consider as zero.
2. Demand is assumed as

$$D_V(P_V, P_R) = \alpha a - bP_V + rP_R$$

$$D_R(P_V, P_R) = ((1 - \alpha)a - bP_R + rP_V) e^{-\mu t}$$

3. The effects of learning on ordering and holding are taken into account for both channels.
4. Time horizon is infinite for the Retailer and the vendor.
5. The market size and deterioration rate do not change with time.
6. Shortages in both channels are not allowed.

V. MATHEMATICAL MODELLING

In this scenario, the vendor acts as the leader in the supply chain. Initially, the vendor procures a large quantity of deteriorating products, denoted by Q_V . These products are then distributed through two channels: to a downstream retailer and directly to end customers via a direct channel. Upon receiving their order, the retailer sells the products to customers through the retail channel. Customers have the flexibility to purchase products either from the direct channel or the retail channel.

As depicted in Fig. 1b, the vendor's inventory depletes due to three main factors: direct channel demand, retail channel orders, and product deterioration. Similarly, the retailer's inventory decreases because of retail channel demand and product deterioration.

Initially, a centralized model is considered, where the vendor and retailer operate as a vertically integrated entity. In this model, there are four key decision variables: the retail price (P_R), the direct channel price (P_V), the vendor's inventory scale parameter (n), and the retailer's ordering cycle length (T).

Subsequently, a decentralized model is examined, where the vendor assumes the role of the Stackelberg leader and the retailer acts as the follower. In this game-theoretic framework, both the vendor and the retailer independently make decisions to maximize their individual profits. The sequence of decision-making unfolds as follows:

1. The vendor sets the wholesale price (ω), the direct sale price (P_V), and the inventory scale parameter (n).
2. The retailer determines the retail price (P_R) and the ordering cycle length (T) based on the vendor's announced decisions.

VI. RETAILER'S PROFIT

$$\frac{dI_{Ri}(t)}{dt} = -\theta I_{Ri}(t) - D_R e^{-\sigma t}, t \in [(i-1)T, iT], i = 1, 2, 3 \dots N \tag{1}$$

$$\text{With boundary condition } I_{Ri}(t = iT) = 0, I_{Ri}[t = (i-1)T] = Q_{Ri} \tag{2}$$

Solution of this eq. with boundary condition is-

$$I_{Ri}(t) = \frac{D_R}{\theta - \sigma} (e^{iT(\theta - \sigma) - \theta t} - e^{-\sigma t}) \text{ and } Q_{Ri} = \frac{D_R}{\theta - \sigma} (e^{(\theta - \sigma i)T} - e^{-\sigma(i-1)T}) \tag{3}$$

Calculated inventory cost for retailer's-

$$\text{Ordering cost- } OC = n(O_R + O'_R + \frac{O''_R}{n\omega}) \tag{4}$$

Holding cost in phase i -

$$HC_{Ri} = (C_{hR} + C'_{hR}) \int_{(i-1)T}^{iT} I_{Ri}(t) e^{-rt} dt = \frac{C_{hR} D_R}{\theta - \sigma} \left(\frac{1}{\theta + r} (e^{T(\theta - i\sigma) + T(1-i)r} - e^{-(\sigma+r)iT}) + \frac{1}{\sigma+r} (e^{-(\sigma+r)iT} - e^{-\sigma(i-1)T - r(i-1)T}) \right) \tag{5}$$

$$\text{Retailer's total holding cost- } HC_R = (C_{hR} + C'_{hR} + \frac{C''_{hR}}{n\omega}) \sum_{i=1}^N HC_{Ri} \tag{6}$$

$$\text{Sales revenue for retailer's- } SR_R = P_R \int_0^{nT} D_R e^{-rt} dt = \frac{P_R D_R}{\sigma+r} (1 - e^{-nT\sigma - rnT}) \tag{7}$$

$$\text{Total purchasing cost- } PC_R = (W + W') \sum_{i=1}^N Q_{Ri} = (W + W') \sum_{i=1}^N \frac{D_R}{\theta - \sigma} (e^{(\theta - \sigma i)T} - e^{-\sigma(i-1)T}) \tag{8}$$

Deterioration cost-

$$DC_{Ri} = (C_{dR} + C'_{dR}) \int_{(i-1)T}^{iT} \theta_1 I_{Ri}(t) e^{-rt} dt = \frac{C_{dR} D_R \theta_1}{\theta - \sigma} \left(\frac{1}{\theta + r} (e^{T(\theta - i\sigma) + T(1-i)r} - e^{-(\sigma+r)iT}) + \frac{1}{\sigma+r} (e^{-(\sigma+r)iT} - e^{-\sigma(i-1)T - r(i-1)T}) \right) \tag{9}$$

$$\text{Total deterioration cost- } DC_R = (C_{dR} + C'_{dR}) \sum_{i=1}^N DC_{Ri} \tag{10}$$

The retailer's total profit per unit of time can be determined using the revenue and cost functions described above-

$$TP_R = \frac{1}{nT} \{SR_R - OC_R - PC_R - HC_R - DC_R\}$$

$$TP_R = \frac{1}{nT} \left\{ \left(\frac{P_R D_R}{\sigma+r} (1 - e^{-nT\sigma - rnT}) \right) - \left(n(O_R + O'_R + \frac{O''_R}{n\omega}) \right) - \left((W + W') \sum_{i=1}^N \frac{D_R}{\theta - \sigma} (e^{(\theta - \sigma i)T} - e^{-\sigma(i-1)T}) \right) - \left((C_{hR} + C'_{hR} + \frac{C''_{hR}}{n\omega}) \sum_{i=1}^N \frac{D_R}{\theta - \sigma} \left(\frac{1}{\theta + r} (e^{T(\theta - i\sigma) + T(1-i)r} - e^{-(\sigma+r)iT}) + \frac{1}{\sigma+r} (e^{-(\sigma+r)iT} - e^{-\sigma(i-1)T - r(i-1)T}) \right) \right) - \left((C_{dR} + C'_{dR}) \sum_{i=1}^N \frac{D_R \theta}{\theta - \sigma} \left(\frac{1}{\theta + r} (e^{T(\theta - i\sigma) + T(1-i)r} - e^{-(\sigma+r)iT}) + \frac{1}{\sigma+r} (e^{-(\sigma+r)iT} - e^{-\sigma(i-1)T - r(i-1)T}) \right) \right) \right\} \tag{11}$$

VII. VENDOR'S PROFIT

$$\frac{dI_{Vi}(t)}{dt} = -\theta I_{Vi}(t) - D_V, t \in [(i-1)T, iT], i = 1, 2, 3 \dots N \tag{12}$$

With boundary conditions

1. $I_{V(i-1)}[t = (i-1)T] - I_{Vi}[t = (i-1)T] = Q_{Ri}, i = 2, 3 \dots N$
 2. $I_{VN}[t = NT] = 0$
 3. $I_{V1}(t = 0) = Q_V - Q_{R1}$
- (13)

The solution of this equation with boundary condition is (see Yong He *et. al.* 2018)

$$I_{Vi}(t) = \frac{D_V}{\theta} (e^{\theta(nT-t)} - 1) + \frac{D_R}{\theta-\sigma} (e^{(\theta-\sigma)nT} - e^{(\theta-\sigma)iT})e^{-\theta t}, t \in [(i-1)T, iT], i = 1, 2 \dots N \quad \text{and} \quad Q_V = \frac{D_V}{\theta} (e^{\theta nT} - 1) + \frac{D_R}{\theta-\mu} (e^{(\theta-\sigma)nT} - 1) \tag{14}$$

Calculated inventory cost for vendor's-

$$\text{Ordering cost- } OC_V = O_V + O'_V + \frac{O''_V}{n\omega} \tag{15}$$

Holding Quantity-

$$HQ_V = \int_{(i-1)T}^{iT} e^{-rt} D_{Vi} dt = \left(\frac{D_V}{\theta} \left(\frac{1}{\theta+r} (e^{T(\theta N - \theta(i-1) - r(i-1))} - e^{T(\theta(N-i) - ri)}) \right) + \frac{1}{r} (e^{-riT} - e^{-r(i-1)T}) - \frac{D_R}{(\theta+r)(\theta-\sigma)} (e^{(\theta-\sigma)NT} - e^{(\theta-\sigma)iT}) (e^{-(\theta+r)iT} - e^{-(\theta+r)(i-1)T}) \right) \tag{16}$$

Vendor's total holding cost-

$$HR_V = C_{hV} HQ_V = \left(C_{hV} + C'_{hV} + \frac{C''_{hV}}{n\omega} \right) \left(\frac{D_V}{\theta} \left(\frac{1}{\theta+r} (e^{T(\theta N - \theta(i-1) - r(i-1))} - e^{T(\theta(N-i) - ri)}) \right) + \frac{1}{r} (e^{-riT} - e^{-r(i-1)T}) - \frac{D_R}{(\theta+r)(\theta-\sigma)} (e^{(\theta-\sigma)NT} - e^{(\theta-\sigma)iT}) (e^{-(\theta+r)iT} - e^{-(\theta+r)(i-1)T}) \right) \tag{17}$$

Vendor's total deterioration cost-

$$DC_V = \theta (C_{dV} + C'_{dV}) \left(\frac{D_V}{\theta} \left(\frac{1}{\theta+r} (e^{T(\theta N - \theta(i-1) - r(i-1))} - e^{T(\theta(N-i) - ri)}) \right) + \frac{1}{r} (e^{-riT} - e^{-r(i-1)T}) - \frac{D_R}{(\theta+r)(\theta-\sigma)} (e^{(\theta-\sigma)NT} - e^{(\theta-\sigma)iT}) (e^{-(\theta+r)iT} - e^{-(\theta+r)(i-1)T}) \right) \tag{18}$$

Vendor's sales revenue-

$$SR_V = P_V D_V NT = P_V A^\gamma ((1-\alpha)a - bP_R + \beta P_V) e^{-\sigma t NT} \tag{19}$$

Whole sales revenue-

$$WR_V = (W + W') \sum_{i=0}^N Q_{Ri} = (W + W') \sum_{i=0}^N \frac{D_R}{\theta-\sigma} (e^{(\theta_1-\sigma i)T} - e^{-\sigma(i-1)T}) \tag{20}$$

$$\text{Vendor's purchasing cost- } PC_V = C_V Q_V = (C_V + C'_V) \left(\frac{D_V}{\theta} (e^{\theta(NT)} - 1) + \frac{D_R}{\theta-\sigma} (e^{(\theta-\sigma)nT} - e^{(\theta-\sigma)iT}) + \frac{D_R}{\theta-\sigma} (e^{(\theta-\sigma)T} - 1) \right) \tag{21}$$

The vendor's total profit per unit of time can be determined using the revenue and cost functions described above-

$$TP_V = \frac{1}{NT} \{SR_V + WR_V - OC_V - HC_V - DC_V - PC_V\}$$

$$\begin{aligned}
 TP_V = \frac{1}{NT} & \left\{ (P_V A^y ((1-\alpha)a - bP_R + \beta P_V) e^{-\sigma t NT}) + \left(W \sum_{i=0}^N \frac{D_R}{\theta-\sigma} (e^{(\theta-\sigma i)T} - e^{-\sigma(i-1)T}) \right) - (O_V + O'_V + \frac{O''_V}{n\omega}) - \left((C_{hV} + \right. \right. \\
 C'_{hV} + \frac{C''_{hV}}{n\omega}) & \left. \left(\frac{D_V}{\theta} \left(\frac{1}{\theta+r} (e^{T(\theta N - \theta(i-1) - r(i-1))} - e^{T(\theta(N-i) - ri)}) \right) + \frac{1}{r} (e^{-riT} - e^{-r(i-1)T}) - \frac{D_R}{(\theta+r)(\theta-\sigma)} (e^{(\theta-\sigma)NT} - \right. \right. \\
 e^{(\theta-\sigma)iT}) & \left. \left. (e^{-(\theta+r)iT} - e^{-(\theta+r)(i-1)T}) \right) \right) - \left(\theta (C_{dV} + C'_{dV}) \left(\frac{D_V}{\theta} \left(\frac{1}{\theta+r} (e^{T(\theta N - \theta(i-1) - r(i-1))} - e^{T(\theta(N-i) - ri)}) \right) + \frac{1}{r} (e^{-riT} - \right. \right. \\
 e^{-r(i-1)T}) & \left. \left. - \frac{D_R}{(\theta+r)(\theta-\sigma)} (e^{(\theta-\sigma)NT} - e^{(\theta-\sigma)iT}) (e^{-(\theta+r)iT} - e^{-(\theta+r)(i-1)T}) \right) \right) - \left((C_V + C'_V) \left(\frac{D_V}{\theta} (e^{\theta NT}) - 1 \right) + \right. \\
 \left. \frac{D_R}{\theta-\sigma} (e^{(\theta-\sigma)NT} - e^{(\theta-\sigma)iT}) + \frac{D_R}{\theta-\sigma} (e^{(\theta-\sigma)T} - 1) \right) \right\} \quad (22)
 \end{aligned}$$

VIII. GRAPHICAL REPRESENTATION

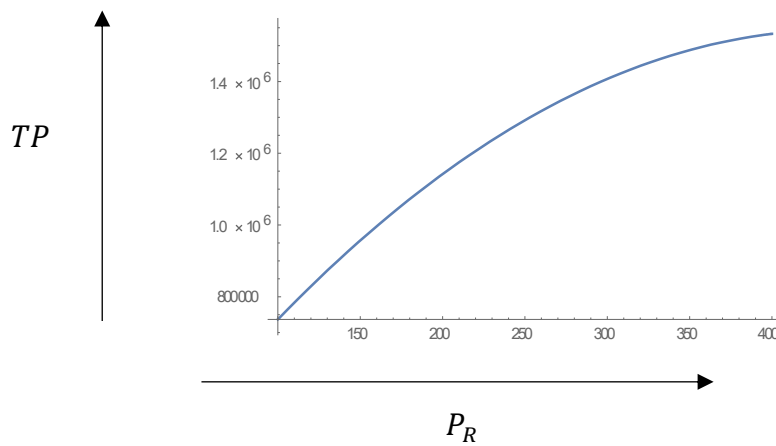


Fig: 3- graphical representation of total profit with respect to P_R (Selling price for retailer channel)

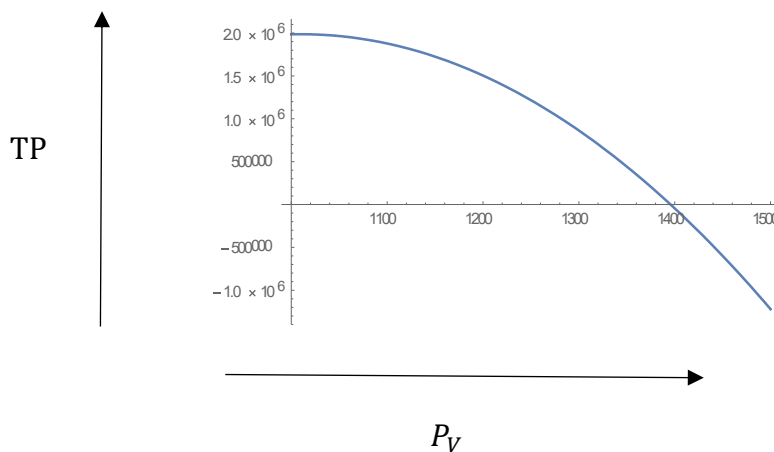


Fig:4- graphical representation of total profit with respect to P_V (Selling price for Vendor channel)

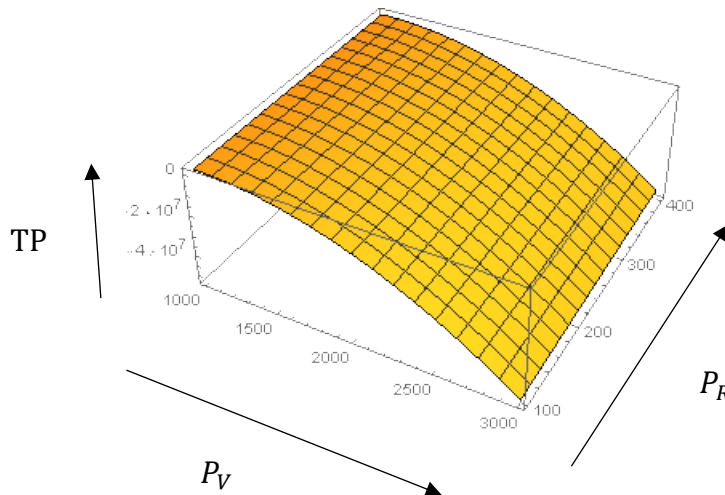


Fig: 5- 3D representation of total profit with respect to P_V (Selling price for Vendor channel) and P_R (Selling price for retailer channel)

IX. NUMERICAL ANALYSIS

To validate the proposed model and assess its practical applicability, a numerical analysis is conducted using realistic data for a dual-channel supply chain with deteriorating products. The analysis examines the effects of key parameters such as product deterioration rate, price sensitivity, advertising effectiveness, learning effect, and inflation on inventory and pricing decisions. The aim is to provide insights into how these factors influence supply chain performance and decision-making. The data values are-

$a=1000, i =, A= 10, \gamma= 1, \alpha= 0.1, \beta= 0.2, b= 0.7, \sigma= 0.1, r= 0.0001, O_R=100, O'_R = 1000, O''_R= 100, O_V= 0.01, O'_V= 5, O''_V= 1100, N= 15, W= 500, W'= 10, C_{hR}= 100, C'_{hR}= 500, \theta= 0.8, C_{dR}= 100, C'_{dR}= 2000, W''= 100, C_{hV}= 0.001, C'_{hV}= 0.02, C''_{hV}= 100, n= 700, \omega= 0.001, C''_{hR}= 7, C_V= 50, C'_V= 8, C_{dV}=0.01, C'_{dV}=10$ and decision variable are $P_V= 1226.26, P_R= 279.039, T=0.0132411$ and total profit is 724365.

X. SENSITIVITY ANALYSIS

The sensitivity analysis is a critical part of the study, as it explores the impact of varying key parameters on the overall performance of the dual-channel supply chain. By systematically altering values such as the product deterioration rate, price elasticity, advertising effectiveness, learning effect, and inflation rate, the sensitivity analysis provides deeper insights into how these factors affect the supply chain's profitability, inventory management, and pricing decisions. So now below table shows how some parameters give impact to decision variable and total profit also.

Table:2

Parameters	% change	P_V	P_R	N	T	Total profit
b	+20%	1198.3	435.983	35.457	1.84932	189234
	+10%	1057.8	372.852	30.1336	1.79214	203424
	0	951.313	171.153	15.4241	0.0147715	228958
	-10%	987.326	167.276	14.2432	0.012345	249721
	-20%	1198.26	153.345	13.2342	0.001438	263267
a	+20%	997.123	233.245	16.2453	1.92345	324643
	+10%	986.345	190.345	24.643	0.96754	288394
	0	951.313	171.153	15.4241	0.0147715	228958
	-10%	945.345	160.24	34.53	0.0345	183245
	-20%	940.244	158.34	25.7433	0.02467	158392
	+20%	1248.34	234.87	29.34	0.00345	110293

γ	+10%	1187.27	467.345	24.53	0.0246	193855
	0	951.313	171.153	15.4241	0.0147715	228958
	-10%	834.738	246.44	13.76	0.2345	273922
	-20%	738.373	324.75	11.346	0.297543	293045
O_V	+20%	823.42	181.34	27.543	0.34432	110293
	+10%	1037.34	176.234	17.42	0.24545	182984
	0	951.313	171.153	15.4241	0.0147715	228958
	-10%	1329.23	169.354	12.42	0.24422	239859
O_R	-20%	826.28	158.43	9.4568	0.13422	259833
	+20%	1129.23	130.44	43.267	0.12344	139220
	+10%	1027.11	169.54	26.356	0.113445	148593
	0	951.313	171.153	15.4241	0.0147715	228958
W	-10%	8234.22	178.45	11.467	0.012455	279483
	-20%	728.345	180.54	6.3589	0.011564	293845
	+20%	1982.33	136.43	10.234	1.45632	237433
	+10%	1523.53	254.98	14.234	1.23554	241834
θ	0	951.313	171.153	15.4241	0.0147715	228958
	-10%	729.345	135.35	16.468	0.012688	219284
	-20%	538.345	243.246	17.346	0.001267	193755
	+20%	234.34	138.234	23.469	0.0011453	168432
r	+10%	892.34	164.246	21.378	0.012353	203853
	0	951.313	171.153	15.4241	0.0147715	228958
	-10%	345.98	245.765	13.753	0.23543	294753
	-20%	424.34	357.46	11.398	0.29535	318303
C_{hR}	+20%	1423.34	332.356	17.409	0.12345	543832
	+10%	1235.33	223.469	25.358	0.16424	438293
	0	951.313	171.153	15.4241	0.0147715	228958
	-10%	753.345	662.43	26.345	0.4245	472974
C_{dR}	-20%	429.567	542.46	29.654	0.46722	247932
	+20%	349.45	125.23	39.75	0.002321	474702
	+10%	587.45	159.34	37.3567	0.003543	247942
	0	951.313	171.153	15.4241	0.0147715	228958
Q_V	-10%	1245.36	184.24	18.654	0.1456	475422
	-20%	1478.34	194.65	23.46	0.193566	457928
	+20%	1937.345	236.33	12.345	0.2356	318293
	+10%	1643.35	376.32	13.456	0.3213	273895
A	0	951.313	171.153	15.4241	0.0147715	228958
	-10%	633.938	464.76	16.467	0.0244	172948
	-20%	356.345	368.24	17.543	0.0042422	147839
	+20%	459.24	349.53	24.643	0.12234	235994
σ	+10%	793.45	653.68	22.977	0.12345	279933
	0	951.313	171.153	15.4241	0.0147715	228958
	-10%	1024.53	254.65	14.468	0.0012334	239344
	-20%	1398.46	127.35	13.456	0.0011356	248945
σ	+20%	1876.46	159.35	35.32	1.4532222	119839
	+10%	1698.57	165.35	26.355	1.24643	224892
	0	951.313	171.153	15.4241	0.0147715	228958
	-10%	689.36	183.46	12.789	0.00243	247903
σ	-20%	528.45	186.35	11.345	0.0015333	294803
	+20%	1532.74	126.45	22.345	2.3422	118033
	+10%	1297.35	148.35	24.567	1.5366	293479
	0	951.313	171.153	15.4241	0.0147715	228958
σ	-10%	590.25	179.345	12.234	0.0129354	546743
	-20%	492.35	210.245	4.246	0.001235	422562

XI. OBSERVATIONS

From the computational results presented in Table 2 following implications are obtained:

1. With increase in parameter 'b' then selling price for vendor is fluctuating, selling price for retailer is increasing, Multiple of retailer's cycle time and total cycle time is also increasing and total profit is decreasing.
2. With increase in parameter 'a' then selling price for vendor, selling price for retailer and total profit are increasing. Multiple of retailer's cycle time and total cycle are fluctuating.
3. With increase in parameter ' γ ' then selling price for vendor and Multiple of retailer's cycle time are increasing while selling price for retailer and total cycle length are fluctuating and total profit is decreasing.
4. With increase in parameter ' O_V ' then selling price for retailer is increasing while selling price for vendor, Multiple of retailer's cycle time and total cycle time are fluctuating and total profit is decreasing.
5. With increase in parameter ' O_R ' then selling price for vendor, Multiple of retailer's cycle time and total cycle time are increasing. Selling price for retailer and total profit are decreasing.
6. With increase in parameter ' W ' then selling price for vendor, total cycle length and total profit are increasing. selling price for retailer and Multiple of retailer's cycle time are decreasing.
7. With increase in parameter ' θ ' then selling price for retailer, total cycle length and total profit are increasing. selling price for vendor is fluctuating while Multiple of retailer's cycle time is decreasing.
8. With increase in parameter ' r ' then selling price for retailer is increasing. selling price for vendor, Multiple of retailer's cycle time, total cycle length and total profit are fluctuating.
9. With increase in parameter ' C_{HR} ' then selling price for retailer, selling price for vendor and total cycle length are decreasing. Multiple of retailer's cycle time is increasing while total profit is fluctuating.
10. With increase in parameter ' C_{dR} ' then selling price for vendor, total cycle length and total profit are increasing. selling price for retailer is fluctuating and Multiple of retailer's cycle time is decreasing.
11. With increase in parameter ' Q_V ' then selling price for vendor is decreasing while selling price for retailer, total profit and Multiple of retailer's cycle are fluctuating. total cycle length is increasing.
12. With increase in parameter ' A ' then selling price for retailer, Multiple of retailer's cycle and total cycle length are increasing while selling price for vendor and total profit are decreasing.
13. With increase in parameter ' σ ' then selling price for vendor, Multiple of retailer's cycle and total cycle length are increasing. selling price for retailer decreasing while total profit is fluctuating.

XII. CONCLUSION

This study develops an integrated model to optimize inventory and pricing decisions for a dual-channel supply chain dealing with deteriorating products, considering demand influenced by price, advertisement, time-dependency, and the learning effect, under inflationary conditions. By incorporating these factors, the model provides a comprehensive framework for managing the complexities of dual-channel operations, especially in industries where products have a limited shelf life and external market conditions are in constant flux. The findings highlight the importance of adaptive pricing and inventory strategies to mitigate the effects of product deterioration and inflation. The sensitivity analysis further demonstrates how key variables such as deterioration rate, price sensitivity, advertising effectiveness, the learning effect, and inflation significantly influence supply chain performance. Results suggest that ignoring these dynamic factors can lead to suboptimal outcomes, such as excess inventory or lost sales, particularly in the presence of deteriorating products. The study also underscores the critical role of the learning effect in reducing operational costs over time, enabling companies to maintain competitive pricing while improving efficiency. Inflation, on the other hand, requires careful management of pricing adjustments to preserve profitability without diminishing demand, particularly in highly price-sensitive environments.

Ultimately, the model and its numerical analysis offer valuable insights for supply chain managers. By understanding how various factors interact, managers can develop more robust strategies to balance profitability and customer satisfaction. This research provides a solid foundation for further exploration of more complex supply chain systems and highlights the potential for integrating additional real-world complexities, such as uncertainty in demand or multi-echelon supply chains, into future studies.

REFERENCES

1. Chiang, W. Y. K., Chhajed, D., & Hess, J. D. (2003). Direct marketing, indirect profits: A strategic analysis of dual-channel supply-chain design. *Management science*, 49(1), 1-20.
2. Yan, R., & Pei, Z. (2009). Retail services and firm profit in a dual-channel market. *Journal of retailing and consumer services*, 16(4), 306-314.
3. Hua, G., Wang, S., & Cheng, T. E. (2010). Price and lead time decisions in dual-channel supply chains. *European journal of operational research*, 205(1), 113-126.
4. Xu, H., Liu, Z. Z., & Zhang, S. H. (2012). A strategic analysis of dual-channel supply chain design with price and delivery lead time considerations. *International Journal of Production Economics*, 139(2), 654-663.

5. Chiang, W. Y. K., Chhajed, D., & Hess, J. D. (2003). Direct marketing, indirect profits: A strategic analysis of dual-channel supply-chain design. *Management science*, 49(1), 1-20.
6. He, Y., Huang, H., & Li, D. (2020). Inventory and pricing decisions for a dual-channel supply chain with deteriorating products. *Operational Research*, 20(3), 1461-1503.
7. Li, M., & Mizuno, S. (2022). Dynamic pricing and inventory management of a dual-channel supply chain under different power structures. *European Journal of Operational Research*, 303(1), 273-285.
8. Das, S., Choudhury, M., Mahato, C., & Mahata, G. C. (2023). Dual-channel supply chain inventory model under two-warehouse setting with order volume-linked trade credit, all-units discount and partial backordering. *Soft Computing*, 27(21), 15817-15852.
9. Darom, N. A., Hishamuddin, H., Ramli, R., & Nopiah, Z. M. (2018). An inventory model of supply chain disruption recovery with safety stock and carbon emission consideration. *Journal of cleaner production*, 197, 1011-1021.
10. Hammami, R., Nouira, I., & Frein, Y. (2015). Carbon emissions in a multi-echelon production-inventory model with lead time constraints. *International Journal of Production Economics*, 164, 292-307.
11. Liu, N., Tang, W., Lan, Y., & Pei, H. (2024). Pricing and carbon reduction decisions for a new uncertain dual-channel supply chain under cap-and-trade regulation. *Fuzzy Optimization and Decision Making*, 23(3), 415-448.