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An EOQ Model for Deteriorating items with Time and Advertisement dependent demand under the Effect of Learning and Inflation

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Abstract—This paper proposes a comprehensive sustainable inventory model tailored for deteriorating items, incorporating multifaceted dynamics such as time, price, and advertisement dependent demand. Amidst the global imperative for sustainability, the model integrates key factors including learning, preservation technology adoption, and the application of carbon cap and trade policy to mitigate carbon emissions. The model is designed to optimize inventory decisions while concurrently addressing environmental concerns and economic viability. By considering the perishable nature of items, demand fluctuations influenced by time, price and advertising effectiveness are incorporated to enhance accuracy in demand forecasting. Preservation technology adoption is integrated as a critical component, offering avenues to extend the shelf life of products and reduce waste. The implementation of carbon cap and trade policy provides a mechanism to internalize environmental costs and incentivize carbon emission reductions within the inventory management framework. Through this policy instrument, firms can strategically allocate emissions allowances, fostering sustainability while maintaining operational efficiency. The proposed model contributes to the sustainability discourse by offering a holistic approach to inventory manage that balances economic objectives with environmental stewardship. Empirical validation and sensitivity analyses demonstrate the efficacy and robustness of the model under various scenarios, highlighting its potential applicability across diverse industries facing similar challenges in managing deteriorating items within a sustainable framework.

Keywords: EOQ Model, demand, economic.

I. INTRODUCTION AND LITERATURE REVIEW

In today's world, it's important to find ways to make our businesses work better while also taking care of our planet. One big challenge is figuring out how to handle products that don't last forever. These products, like food or medicine, can go bad over time, and it's not easy to decide how much of them to keep in stock. This paper talks about a new idea for dealing with this challenge. Instead of just guessing how much of these products to keep, we're looking at lots of different factors that affect how much people want to buy them. Things like when people tend to buy more, how the price affects sales, and how advertising can make a difference and also thinking about how technology can help. There are ways to make products last longer, and we want to see how using these technologies can help us waste less and be more sustainable. Here considering how can help the environment by reducing carbon emissions and suggesting a plan where companies have to limit the amount of carbon they emit, and they can trade with each other to stay within those limits. This way, companies have a reason to find greener ways to do things. By looking at all of these things together, we hope to find a way to run businesses that makes sense economically and helps us take care of the environment at the same time. In this manner Panda *et al.* (2008) developed model in which they considered stock-dependent demand with inventory control policy and give discount on price when deterioration period is start. Chen et al. (2013) discovered that operational adjustments could significantly reduce carbon footprints. As a result, carbon footprints were reduced while remaining relatively unchanged in cost. They evaluated various environmental regulations such as the carbon tax, cap and offset, carbon caps, and cap and price to determine the best solution. Shah *et al.* (2013) considered inventory model for non-instantaneous deterioration products and demand is depend on price and advertisement. Shastri *et al.* (2014) investigated a model where demand is depended on selling price and also give some theoretical results for optimal solution. Palanivel *et al.* (2015) presented an economic order quantity model with selling and advertisement depended demand and also considered inflationary environment. Zhang *et al.* (2015) investigated a EOQ model with preservation technology to control deteriorated items. Yadav *et al.* (2016) developed a EOQ model with multi-item and multi-constrained integrated model and also adopt preservation technology. Xu et al. (2017) analysed the integrated problem's production and carbon emission reduction decisions while taking green technology investment into consideration. Numerically, they discovered that increasing the carbon trading price causes the optimal production quantity to first increase and then stabilize. Mishra (2018) developed production model where shortages were completely backlogged and demand depended on price and time. Tiwari et al. (2019) presented an inventory control policy for non-instantaneous decomposing items in an environment of uncertainty to reduce carbon emissions while increasing the system's annual profit. Shaikh *et al.* (2019) investigated two model with zero ending stock and non-zero ending stock levels and partial backlogging is also considered. Taleizadeh et al. (2020) developed a model with carbon emission control policy and allowing shortages which is partially backlogged. Sepehri et al. (2021) analyzed the production model to reduce carbon emissions by investing in carbon-reduction technology and improve the product's quality by investing in a quality improvement program. Sarkar et al. (2021) developed the best policy for a sustainable supply chain to control the carbon footprint and product quality. They used a specific algebraic method to arrive at the optimal policy. San-jose *et al.* (2021) developed a model in which they considered price, time and advertisement dependent demand and also give an algorithm for optimal solution. They also considered partial backlogged and holding cost have two components one is fixed and other is variable. Mishra and Mishra (2022) considered a sustainable inventory model with carbon emissions and non-instantaneous also considered partial backlogging. Sarkar et al. (2022) developed a remanufacturing model for green innovative products with uncertain return and demand rates, aiming to reduce environmental waste. Kugele et al. (2022) designed a smart production system to control carbon ejection and use geometric programming to find the best solution. To achieve environmental sustainability, the current study considered capital investment in green technology, carbon footprint caps, and a tax on emitted carbon footprints. Wei et al. (2024) established a two-period production and selling model using a dynamic pricing strategy and a price commitment strategy, respectively, while taking into account the stochastic learning effect and whether or not the firm has the inventory carryover option. There is no such model in this existing literature that has price, time and advertisement depended demand with inflation and preservation technology to control deteriorated item and also adopt policy to control carbon emissions.

II. ASSUMPTIONS AND NOTATIONS

The following assumptions are considering in this model:

1. The demand function is dependent on time, selling price and advertisement cost.
$$D(p,t,\vartheta) = (a - bp + ct)\vartheta^{\mu}$$
2. Under the effect of preservation technology, deterioration is of the form:
$$\theta(\phi) = \big(1 - m(\phi)\big)\theta$$
3. Carbon emissions due to some activities are considering and to reduce carbon footprint, investment in green technology is considering and investment in green technology carbon emissions is reduced by $\big(1 - \lambda(1 - e^{-\sigma G_T})\big)$.
4. Shortage is allowed partial backlogged with the backlogging rate $\delta(x) = e^{-\varepsilon x}$.
5. Inflation is also considering.
6. Effect of learning on ordering, holding and disposal costs are considering.

Parameter	
a	Constant
b	Constant
p	Selling price
ϑ	Advertisement cost
μ	Constant
φ	Investment in preservation technology
ψ	Constant
θ	Rate of deterioration
Q_1	Inventory at t=0
C_T	Carbon tax
U_c	Upper cap of carbon emissions
C_p	Purchasing cost
B_1	Maximum backorder level
ϵ	Backlogging parameter
K	Ordering cost
K'	Ordering
C_{h_1}	Holding cost
$C_{h_1}{}'$	Holding cost
C_D	Deterioration cost
r	Rate of inflation
C_{DC}	Disposable cost
$C_{DC}{}'$	Disposable cost
n	Cumulative frequency of shipment
ω	Learning rate
C_s	Shortage cost

C_l	Lost sale cost
A_d	Advertisement cost
$A_d{}'$	Advertisement cost
B	Carbon cost associated due placing order
P	Carbon cost due to per unit purchase
H	Carbon cost due to per unit holding
D	Carbon cost due to per unit disposable
κ	Maximum amount of carbon emission
Decision variable	
T	Cycle time
p	Selling price
t_1	Time where shortage start

III. MATHEMATICAL MODELLING

The retailer placed order that fulfils customer demand and for backlogging the shortage quantity under cap-and-trade policy. Its model is shown in the below Fig.1. The initial inventory level Q_1 is decreasing due to demand and deterioration. At the point t_1, the inventory become 0, and after that, shortages occur and accumulate up to the time T.

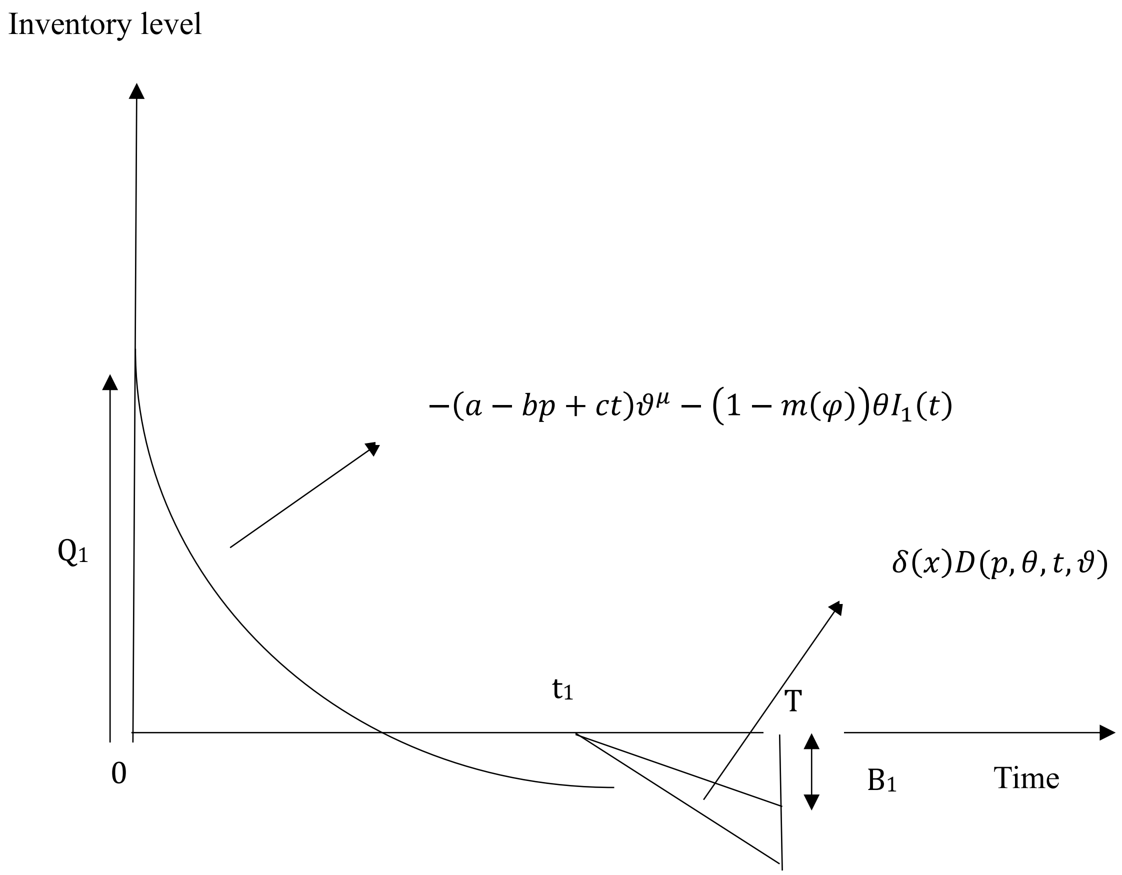

Fig:1-Graphical representation of sustainable inventory model

The differential equations of the given model are represented by:

$$\frac{dI_1(t)}{dt}=-(a-bp+ct)\vartheta^{\mu}-\big(1-m(\varphi)\big)\theta I_1(t), 0\le t\le t_1.$$

$$\frac{dI_2(t)}{dt}=-\delta(x)(a-bp+ct)\vartheta^{\mu}, t_1\le t\le T.$$

$$with\ condition\ I_1(t_1)=0, I_1(0)=Q_1, I_2(T)=-B_1$$

After solving these equations, we get

$$I_1(t) = \left(1 - e^{(1-m(\varphi))\theta(t_1-t)}\right)\left(\frac{(a-bp)\vartheta^{\mu}}{(1-m(\varphi))\theta} + \frac{\vartheta^{\mu}c}{\left((1-m(\varphi))\theta\right)^2}\right) - \frac{\vartheta^{\mu}c}{(1-m(\varphi))\theta}\left(t - t_1 e^{(1-m(\varphi))\theta(t_1-t)}\right)$$

and

$$Q_1 = \left(1 - e^{(1-m(\varphi))\theta t_1}\right)\left(\frac{(a-bp)\vartheta^{\mu}}{(1-m(\varphi))\theta} + \frac{\vartheta^{\mu}c}{\left((1-m(\varphi))\theta\right)^2}\right) + \frac{\vartheta^{\mu}c}{(1-m(\varphi))\theta} t_1 e^{-(1-m(\varphi))\theta t}$$

$$I_2(t) = -(a-bp)\vartheta^{\mu}\frac{e^{-\epsilon(T-t)}}{\epsilon} - c\vartheta^{\mu}\left(t\frac{e^{-\epsilon(T-t)}}{\epsilon} - \frac{e^{-\epsilon(T-t)}}{\epsilon^2}\right) + \frac{(a-bp)\vartheta^{\mu}}{\epsilon} + c\vartheta^{\mu}\left(\frac{T}{\epsilon} - \frac{1}{\epsilon^2}\right) - B_1$$

And $B_1 = \frac{\vartheta^{\mu}(a-bp)}{\epsilon}\left(1 - \frac{e^{-(\epsilon(T-t1))}}{\epsilon}\right) - c\vartheta^{\mu}e^{-(\epsilon(T-t1))}\left(\frac{t_1}{\epsilon} - \frac{1}{\epsilon^2}\right) + c\,\vartheta^{\mu}\left(\frac{T}{\epsilon} - \frac{1}{\epsilon^2}\right)$

Now we are evaluating the different costs:

Sales revenue-

$$SR = p\left(\int_0^{t_1}(a - bp + ct)\,\vartheta^{\mu}dt + B_1\right)$$

$$= p\left[(a-bp)\vartheta^{\mu}t_1 + c\vartheta^{\mu}t - \frac{t_1^2}{2} + B_1\right]$$

Carbon Allowance Cost $=C_T U_c$

Purchasing Cost = $C_p(Q_1 + B_1)$

$$PC = C_p\left[\left(\left(1 - e^{(1-m(\varphi))\theta t_1}\right)\left(\frac{(a-bp)\vartheta^{\mu}}{(1-m(\varphi))\theta} + \frac{\vartheta^{\mu}c}{\left((1-m(\varphi))\theta\right)^2}\right) + \frac{\vartheta^{\mu}c}{(1-m(\varphi))\theta}t_1 e^{-(1-m(\varphi))\theta t}\right) + \left(\frac{\vartheta^{\mu}(a-bp)}{\epsilon}\left(1 - \frac{e^{-(\epsilon(T-t1))}}{\epsilon}\right) - c\vartheta^{\mu}e^{-(\epsilon(T-t1))}\left(\frac{t_1}{\epsilon} - \frac{1}{\epsilon^2}\right) + c\,\vartheta^{\mu}\left(\frac{T}{\epsilon} - \frac{1}{\epsilon^2}\right)\right)\right]$$

Ordering Cost$=K + \frac{K'}{n^{\omega}}$

Holding cost$=\int_0^{t_1}\left[e^{-rt}\left(C_{h_1} + \frac{C'_{h_1}}{n^{\omega}}\right) + te^{-rt}\left(C_{h_2} + \frac{C'_{h_2}}{n^{\omega}}\right)\right]I_1(t)dt$

$$HC = \left(C_{h_1} + \frac{C'_{h_1}}{n^{\omega}}\right)\left[\left(\frac{(a-bp)\vartheta^{\mu}}{(1-m(\varphi))\theta} + \frac{\vartheta^{\mu}c}{\left((1-m(\varphi))\theta\right)^2}\right)\left(\frac{1-e^{-rt}}{r} + \frac{e^{-rt}-e^{(1-m(\varphi))\theta t_1}}{(1-m(\varphi))\theta + r}\right) - \frac{\vartheta^{\mu}c}{(1-m(\varphi))\theta}\left(\left(\frac{1}{r^2} - \frac{e^{-rt}}{r^2} - \frac{t_1e^{-rt}}{r}\right) + t_1\left(\frac{e^{(1-m(\varphi))\theta t_1} - e^{-rt}}{(1-m(\varphi))\theta + r}\right)\right)\right] + \left(C_{h_2} + \frac{C'_{h_2}}{n^{\omega}}\right)\left[\left(\frac{(a-bp)\vartheta^{\mu}}{(1-m(\varphi))\theta} + \frac{\vartheta^{\mu}c}{\left((1-m(\varphi))\theta\right)^2}\right)\left(-\frac{t_1e^{-rt}}{r} - \frac{e^{-rt}}{r^2} + \frac{t_1e^{-rt}}{(1-m(\varphi))\theta + r} + \frac{e^{-rt}}{\left((1-m(\varphi))\theta + r\right)^2} + \frac{1}{r^2} - \frac{e^{(1-m(\varphi))\theta t_1}}{\left((1-m(\varphi))\theta + r\right)^2}\right) - \frac{\vartheta^{\mu}c}{(1-m(\varphi))\theta}\left(-\frac{t_1^2e^{-rt}}{r} - 2t_1\frac{e^{-rt}}{r^2} - \frac{2e^{-rt}}{r^3} + \frac{2}{r^3}\right) - t_1\left(-\frac{t_1e^{-rt}}{(1-m(\varphi))\theta - r} - \frac{e^{-rt}}{\left((1-m(\varphi))\theta + r\right)^2} + \frac{e^{(1-m(\varphi))\theta t_1}}{\left((1-m(\varphi))\theta + r\right)^2}\right)\right]$$

Deterioration cost$=C_D\int_0^{t_1}[e^{-rt}]I_1(t)dt$

$$DC = C_D\left[\left(\frac{(a-bp)\vartheta^{\mu}}{(1-m(\varphi))\theta} + \frac{\vartheta^{\mu}c}{\left((1-m(\varphi))\theta\right)^2}\right)\left(\frac{1-e^{-rt}}{r} + \frac{e^{-rt}-e^{(1-m(\varphi))\theta t_1}}{(1-m(\varphi))\theta + r}\right) - \frac{\vartheta^{\mu}c}{(1-m(\varphi))\theta}\left(\left(\frac{1}{r^2} - \frac{e^{-rt}}{r^2} - \frac{t_1e^{-rt}}{r}\right) + t_1\left(\frac{e^{(1-m(\varphi))\theta t_1} - e^{-rt}}{(1-m(\varphi))\theta + r}\right)\right)\right]$$

Disposable Cost $=\left(C_{DC} + \frac{C'_{DC}}{n^{\omega}}\right)\left(Q_1 - \int_0^{t_1}(a-bp+ct)\vartheta^{\mu}dt\right)$

$$DC = \left(C_{DC} + \frac{C'_{DC}}{n^{\omega}}\right)\left[Q_1 - (a-bp)\vartheta^{\mu}t_1 - \frac{ct_1^2}{2}\vartheta^{\mu}\right]$$

Shortage cost$=C_s\int_{t_1}^{T} -I_2(t)dt$

$$SC = -C_s\left[\frac{(a-bp)\vartheta^{\mu}}{\epsilon^2}\left(1-e^{-\epsilon(T-t_1)}\right) + c\vartheta^{\mu}\left(\frac{T}{\epsilon^2} - \frac{2}{\epsilon^3} - \frac{t_1e^{-\epsilon(T-t_1)}}{\epsilon^2} + \frac{2e^{-\epsilon(T-t_1)}}{\epsilon^3}\right) - \frac{(a-bp)\vartheta^{\mu}}{\epsilon}(T-t_1) - c\vartheta^{\mu}\left(\frac{T}{\epsilon} - \frac{1}{\epsilon^2}\right)(T-t_1) + B_1(T-t_1)\right]$$

Lost sale Cost$=C_l\int_{t_1}^{T}\left(1-e^{-\epsilon(T-t)}\right)(a-bp+ct)\vartheta^{\mu}dt$

$$LSC = C_l[(a-bp)\vartheta^{\mu}(T-t_1) + \frac{\vartheta^{\mu}c(T^2-t_1^2)}{2} - \frac{\vartheta^{\mu}(a-bp)\left(1-e^{-\epsilon(T-t_1)}\right)}{\epsilon} - \vartheta^{\mu}c\left(\frac{T}{\epsilon} - \frac{1}{\epsilon^2} - t_1\frac{e^{-\epsilon(T-t_1)}}{\epsilon} + \frac{e^{-\epsilon(T-t_1)}}{\epsilon^2}\right)]$$

Preservation Technology Investment $=\varphi$

Advertisement Cost$=A_d + \frac{A'_d}{n^{\omega}}$

Green Technology Cost$=G_T$

Carbon Tax- Activities like placing the order, purchasing the items, holding the items in stock, and disposing of the items cause the emission of carbon.

1. Carbon emission due to Placing the order=B
2. Carbon emission due to purchasing the items=$P(Q_1 + B_1)$
3. Carbon emission due to holding the items = $H\left[\left(\frac{(a-bp)\vartheta^{\mu}}{(1-m(\varphi))\theta} + \frac{\vartheta^{\mu}c}{\left((1-m(\varphi))\theta\right)^2}\right)\left(\frac{1-e^{-rt}}{r} + \frac{e^{-rt}-e^{(1-m(\varphi))\theta t_1}}{(1-m(\varphi))\theta+r}\right) - \frac{\vartheta^{\mu}c}{(1-m(\varphi))\theta}\left(\left(\frac{1}{r^2} - \frac{e^{-rt}}{r^2} - \frac{t_1e^{-rt}}{r}\right) + t_1\left(\frac{e^{(1-m(\varphi))\theta t_1}-e^{-rt}}{(1-m(\varphi))\theta+r}\right)\right)\right]$
4. Carbon emission due to disposing of the item=$D\left[Q_1 - (a-bp)\vartheta^{\mu}t_1 - \frac{ct_1^2}{2}\vartheta^{\mu}\right]$

So total tax paid by retailer to the regulatory authority under the effect of investment in green technology is $CE = \left[B + P(Q_1 + B_1) + H\left[\left(\frac{(a-bp)\vartheta^{\mu}}{(1-m(\varphi))\theta} + \frac{\vartheta^{\mu}c}{\left((1-m(\varphi))\theta\right)^2}\right)\left(\frac{1-e^{-rt}}{r} + \frac{e^{-rt}-e^{(1-m(\varphi))\theta t_1}}{(1-m(\varphi))\theta+r}\right) - \frac{\vartheta^{\mu}c}{(1-m(\varphi))\theta}\left(\left(\frac{1}{r^2} - \frac{e^{-rt}}{r^2} - \frac{t_1e^{-rt}}{r}\right) + t_1\left(\frac{e^{(1-m(\varphi))\theta t_1}-e^{-rt}}{(1-m(\varphi))\theta+r}\right)\right)\right] + D\left[Q_1 - (a-bp)\vartheta^{\mu}t_1 - \frac{ct_1^2}{2}\vartheta^{\mu}\right]\right](1-\lambda(1-e^{-\sigma G_T}))$

On adding all these costs, we get the total inventory cost. Now, the total profit of the system is

$$TP = SR - PC - CAC - OC - HC - DC - DIC - SC - LC - PTI - AC - GTC + C_T(\kappa - CE)$$

$$= p\left[(a-bp)\vartheta^{\mu}t_1 + c\vartheta^{\mu}t - \frac{t_1^2}{2} + B_1\right] - [C_TU_c] - \left[C_p\left[\left(\left(1 - e^{(1-m(\varphi))\theta t_1}\right)\left(\frac{(a-bp)\vartheta^{\mu}}{(1-m(\varphi))\theta} + \frac{\vartheta^{\mu}c}{\left((1-m(\varphi))\theta\right)^2}\right) + \frac{\vartheta^{\mu}c}{(1-m(\varphi))\theta}t_1e^{-(1-m(\varphi))\theta t}\right) + \left(\frac{\vartheta^{\mu}(a-bp)}{\epsilon}\left(1 - \frac{e^{-(\epsilon(T-t1))}}{\epsilon}\right) - c\vartheta^{\mu}e^{-(\epsilon(T-t1))}\left(\frac{t_1}{\epsilon} - \frac{1}{\epsilon^2}\right) + c\,\vartheta^{\mu}\left(\frac{T}{\epsilon} - \frac{1}{\epsilon^2}\right)\right)\right]\right] - \left[K + \frac{K'}{n^{\omega}}\right]$$

$$- \left[\left(C_{h_1} + \frac{C'_{h_1}}{n^{\omega}}\right)\left[\left(\frac{(a-bp)\vartheta^{\mu}}{(1-m(\varphi))\theta} + \frac{\vartheta^{\mu}c}{\left((1-m(\varphi))\theta\right)^2}\right)\left(\frac{1-e^{-rt}}{r} + \frac{e^{-rt}-e^{(1-m(\varphi))\theta t_1}}{(1-m(\varphi))\theta+r}\right) - \frac{\vartheta^{\mu}c}{(1-m(\varphi))\theta}\left(\left(\frac{1}{r^2} - \frac{e^{-rt}}{r^2} - \frac{t_1e^{-rt}}{r}\right) + t_1\left(\frac{e^{(1-m(\varphi))\theta t_1}-e^{-rt}}{(1-m(\varphi))\theta+r}\right)\right)\right] + \right.$$

$$\left(C_{h_2} + \frac{C'_{h_2}}{n^{\omega}}\right)\left[\left(\frac{(a-bp)\vartheta^{\mu}}{(1-m(\varphi))\theta} + \frac{\vartheta^{\mu}c}{\left((1-m(\varphi))\theta\right)^2}\right)\left(-\frac{t_1e^{-rt}}{r} - \frac{e^{-rt}}{r^2} + \frac{t_1e^{-rt}}{(1-m(\varphi))\theta+r} + \frac{e^{-rt}}{\left((1-m(\varphi))\theta+r\right)^2} + \frac{1}{r^2} - \frac{e^{(1-m(\varphi))\theta t_1}}{\left((1-m(\varphi))\theta+r\right)^2}\right) -\right.$$

$$\left.\left.\frac{\vartheta^{\mu}c}{(1-m(\varphi))\theta}\left(-\frac{t_1^2e^{-rt}}{r} - 2t_1\frac{e^{-rt}}{r^2} - \frac{2e^{-rt}}{r^3} + \frac{2}{r^3}\right) - t_1\left(-\frac{t_1e^{-rt}}{(1-m(\varphi))\theta-r} - \frac{e^{-rt}}{\left((1-m(\varphi))\theta+r\right)^2} + \frac{e^{(1-m(\varphi))\theta t_1}}{\left((1-m(\varphi))\theta+r\right)^2}\right)\right]\right]$$

$$- \left[C_D\left[\left(\frac{(a-bp)\vartheta^{\mu}}{(1-m(\varphi))\theta} + \frac{\vartheta^{\mu}c}{\left((1-m(\varphi))\theta\right)^2}\right)\left(\frac{1-e^{-rt}}{r} + \frac{e^{-rt}-e^{(1-m(\varphi))\theta t_1}}{(1-m(\varphi))\theta+r}\right) - \frac{\vartheta^{\mu}c}{(1-m(\varphi))\theta}\left(\left(\frac{1}{r^2} - \frac{e^{-rt}}{r^2} - \frac{t_1e^{-rt}}{r}\right) + t_1\left(\frac{e^{(1-m(\varphi))\theta t_1}-e^{-rt}}{(1-m(\varphi))\theta+r}\right)\right)\right]\right] - \left[\left(C_{DC} + \frac{C'_{DC}}{n^{\omega}}\right)\left[Q_1 - (a-bp)\vartheta^{\mu}t_1 - \frac{ct_1^2}{2}\vartheta^{\mu}\right]\right]$$

$$- \left[-C_s\left[\frac{(a-bp)\vartheta^{\mu}}{\epsilon^2}\left(1 - e^{-\epsilon(T-t_1)}\right) + c\vartheta^{\mu}\left(\frac{T}{\epsilon^2} - \frac{2}{\epsilon^3} - \frac{t_1e^{-\epsilon(T-t_1)}}{\epsilon^2} + \frac{2e^{-\epsilon(T-t_1)}}{\epsilon^3}\right) - \frac{(a-bp)\vartheta^{\mu}}{\epsilon}(T-t_1) - c\vartheta^{\mu}\left(\frac{T}{\epsilon} - \frac{1}{\epsilon^2}\right)(T-t_1) + B_1(T-t_1)\right]\right]$$

$$- \left[C_l\left[(a-bp)\vartheta^{\mu}(T-t_1) + \frac{\vartheta^{\mu}c(T^2-t_1^2)}{2} - \frac{\vartheta^{\mu}(a-bp)(1-e^{-\epsilon(T-t_1)})}{\epsilon} - \vartheta^{\mu}c\left(\frac{T}{\epsilon} - \frac{1}{\epsilon^2} - t_1\frac{e^{-\epsilon(T-t_1)}}{\epsilon} + \frac{e^{-\epsilon(T-t_1)}}{\epsilon^2}\right)\right]\right] - [\varphi] - \left[A_d + \frac{A'_d}{n^{\omega}}\right] - [G_T]$$

$$+ C_T\left(\kappa - \left(\left[B + P(Q_1 + B_1) + H\left[\left(\frac{(a-bp)\vartheta^{\mu}}{(1-m(\varphi))\theta} + \frac{\vartheta^{\mu}c}{\left((1-m(\varphi))\theta\right)^2}\right)\left(\frac{1-e^{-rt}}{r} + \frac{e^{-rt}-e^{(1-m(\varphi))\theta t_1}}{(1-m(\varphi))\theta+r}\right) - \frac{\vartheta^{\mu}c}{(1-m(\varphi))\theta}\left(\left(\frac{1}{r^2} - \frac{e^{-rt}}{r^2} - \frac{t_1e^{-rt}}{r}\right) + t_1\left(\frac{e^{(1-m(\varphi))\theta t_1}-e^{-rt}}{(1-m(\varphi))\theta+r}\right)\right)\right] + D\left[Q_1 - (a-bp)\vartheta^{\mu}t_1 - \frac{ct_1^2}{2}\vartheta^{\mu}\right]\right](1-\lambda(1-e^{-\sigma G_T}))\right)\right)$$

IV. NUMERICAL ANALYSIS

Parameter	Values	Parameter	Values
a	100	A'	1
b	8	C_t	0.2 per kg
ϑ	0.1	C_d	10 $
μ	0.02	C_{dc}	0.4 $
θ	0.001 unit per week	C_{dc}'	0.1 $
ϵ	0.021	n	5
λ	0.4	ω	0.023
φ	10	r	0.04
c	1	C_l	0.046 $
C_{h_1}	0.03 per week	K	16 $
C_{h_1}'	0.1 per week	K'	0.4 $
C_{h_2}	0.03 per week	G_T	1 $
C_{h_2}'	0.02 per week	C_s	0.2 $
ψ	0.02	D	1 kg
A	1	C_p	6 $
σ	0.03	H	5 kg
P	0.27 kg	B	10 kg

Solution-

Total profit	553474 rupees
Complete cycle length	3107.76 days
Time when shortage start	3306.7 days
Selling price	0.8173 $

V. SENSITIVITY ANALYSIS

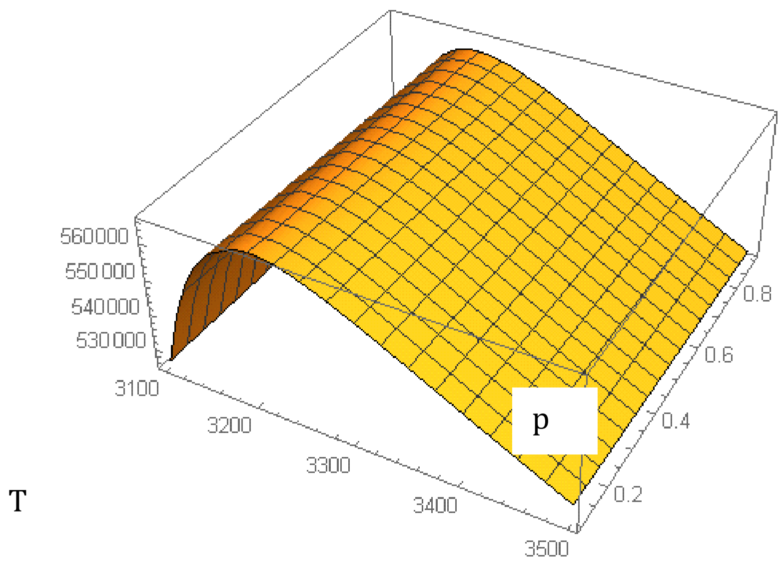

Fig: 2- concavity with respect to T and p

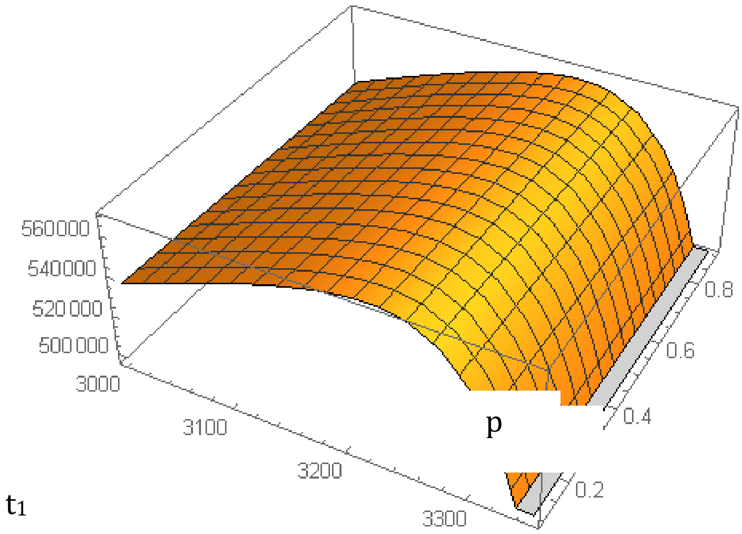

Fig: 3- concavity with respect to t_1 and p

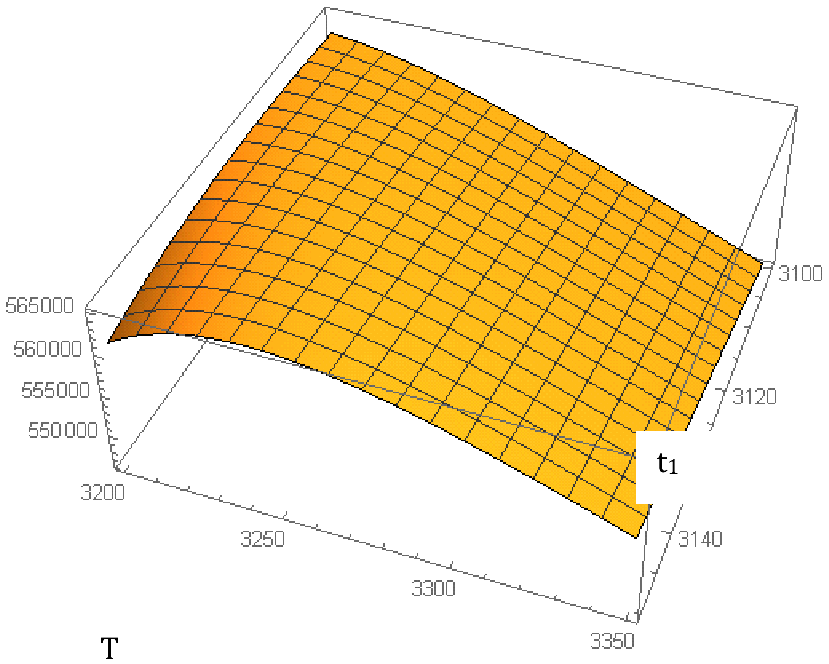

Fig: 4- concavity with respect to T and t_1

Optimal values of decision variables and total profit for different values of different parameters:

Parameter	% change	t_1	T	p	Total profit
a	+20 %	3324.5	3423.53	0.817369	581003
	+10 %	3321.6	3418.44	0.817368	575974

	-10 %	3315.64	3407.65	0.817369	565972
	-20 %	3312.57	3401.87	0.817369	561004
b	+20 %	3318.65	3413.16	0.817369	570963
	+10 %	3318.65	3413.16	0.817368	570963
	-10 %	3318.65	3413.16	0.817369	570963
	-20 %	3318.65	3413.16	0.817369	570963
ϑ	+20 %	3107.76	3306.7	0.817369	555495
	+10 %	3107.76	3306.7	0.817368	554530
	-10 %	3107.76	3306.7	0.817369	552310
	-20 %	3107.76	3306.7	0.817369	551011
μ	+20 %	3107.76	3306.7	0.817369	548403
	+10 %	3107.76	3306.7	0.817369	550933
	-10 %	3107.76	3306.7	0.817369	556028
	-20 %	3107.76	3306.7	0.817369	558592
θ	+20 %	2781.44	2880.77	0.817369	485542
	+10 %	3025.6	3122.59	0.817368	524398
	-10 %	3676.91	3768.77	0.817369	627801
	-20 %	3676.91	3768.77	0.817369	627801
φ	+20 %	3451.09	3544.56	0.817369	592083
	+10 %	3384.2	3478.19	0.817368	581419
	-10 %	3254.39	3349.43	0.817369	560711
	-20 %	3191.41	3286.97	0.817369	550660
c	+20 %	3296.2	3386.88	0.817369	667782
	+10 %	3306.41	3398.89	0.817368	619353
	-10 %	3333.62	3430.41	0.817369	522625
	-20 %	3352.36	3451.74	0.817369	474355
A	+20 %	3107.76	3306.7	0.817369	553474
	+10 %	3107.76	3306.7	0.817369	553474
	-10 %	3107.76	3306.7	0.817369	553474
	-20 %	3107.76	3306.7	0.817369	553474
G_T	+20 %	3323.56	3417.98	0.817369	572310
	+10 %	3321.11	3415.57	0.817368	571637
	-10 %	3317.12	3410.50	0.817369	562306
	-20 %	3313.72	3408.32	0.817369	569613
B	+20 %	3107.76	3306.7	0.817369	553474
	+10 %	3107.76	3306.7	0.817369	553474
	-10 %	3107.76	3306.7	0.817369	553474
	-20 %	3107.76	3306.7	0.817369	553474
A'	+20 %	3451.09	3417.98	0.817369	548403
	+10 %	3384.25	3415.57	0.817368	550933
	-10 %	3254.39	3410.50	0.817369	556028
	-20 %	3191.41	3408.32	0.817369	558592
C_p	+20 %	3107.76	3386.88	0.817369	572310
	+10 %	3107.76	3398.89	0.817368	571637
	-10 %	3107.76	3430.41	0.817369	562306
	-20 %	3107.76	3451.74	0.817369	569613
C_{dc}	+20 %	3323.56	3306.7	0.817369	570963
	+10 %	3321.11	3306.7	0.817368	570963
	-10 %	3317.12	3306.7	0.817369	570963
	-20 %	3313.72	3306.7	0.817369	570963

On increasing in parameter a, cycle length, t_1, and total profit increases while selling price fluctuating. On increasing in parameter b, t_1 and cycle length are constant while selling price fluctuating and total profit decreases. On increasing in parameter ϑ, t_1 and cycle length are constant while selling price fluctuating and total profit increases. On increasing in parameter μ, t_1 and cycle length and selling price are constant while total profit is fluctuating. On increasing in parameter θ, t_1 and cycle length and total profit are decreasing while selling price fluctuating. On increasing in parameter φ, t_1 and total profit are increasing while total cycle length decreasing and selling price is fluctuating. On increasing in parameter c, t_1 and cycle length are decreasing while selling price is fluctuating and total profit increasing. On increasing in parameter A, t_1, cycle length, selling price and total profit are all constant. On increasing in parameter G_T, t_1, cycle length and total profit are increasing while selling price is fluctuating. On increasing in parameter B, t_1, cycle length, selling price and total profit are all constant. On increasing in parameter A', t_1, cycle length and total profit are increasing while selling price is fluctuating. On increasing in parameter C_p, t_1 is constant, cycle length is increasing, selling price fluctuating and total profit is decreasing. On increasing in parameter C_{dc}, $t_1$is increasing, cycle length and total profit are decreasing while selling price fluctuating.

VI. CONCLUSION

This research has explored a new approach to handling products that don't last forever in a way that's good for both businesses and the environment. By considering factors like when people buy more, how prices and advertising affect sales, and how technology can help products last longer, we've developed a better way to manage inventory. We've also suggested using a system where companies have to limit the amount of carbon they emit and can trade with each other to stay within those limits. This gives them a reason to find greener ways to do things, which helps protect the environment. This model can be extended with two or more warehouses, fuzzy can be considered, trade credit and smart production system also taken. This model work shows that it's possible to run businesses in a way that's smart economically and helps us take care of the planet. By using these ideas, we can build a more sustainable future for everyone.

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