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A Two Ware-houses Inventory Model with Different Deterioration rate under Learning Effect

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Abstract— This research paper a thorough investigation into the dynamics of inventory management within dual warehouse systems, focusing on the intricate interaction between linear and nonlinear deterioration rates. By examining various deterioration conditions and integrating considerations of carbon emissions, inflation, and dynamic demand dependencies on both price and time, a holistic understanding of inventory degradation processes is achieved. Through the utilization of mathematical modelling and simulation methodologies, the study explores the implications of different deterioration scenarios on inventory turnover and environmental sustainability. Additionally, it investigates the effects of carbon emissions and inflation on inventory management strategies, highlighting the necessity for adaptive approaches to mitigate adverse impacts. Moreover, by incorporating dynamic demand dependencies on price and time, the research sheds light on the complex interplay between pricing strategies, demand fluctuations, and inventory management decisions. The findings contribute to both theoretical advancements in inventory management and practical insights for optimizing operations within dual warehouse systems amidst evolving environmental and economic dynamics.

Keywords- two-warehouses, carbon emission, inflation, partial backlogging

I. INTRODUCTION

Inventory management is vital for organizational success, particularly within dual warehouse systems designed to bolster supply chain resilience. Understanding deterioration rates, whether linear or nonlinear, is fundamental to optimizing inventory control. However, contemporary inventory paradigms must also grapple with environmental and economic pressures. Carbon emissions and inflationary factors impact sustainability and cost structures, while dynamic demand dependencies complicate inventory allocation. This study aims to comprehensively analyse these factors within dual warehouse systems, offering insights for practitioners to navigate complexities effectively. Through mathematical modelling and empirical analysis, the research seeks to advance both theoretical understanding and practical application, ultimately enhancing operational efficiency, sustainability, and profitability. Several researchers have examined inventory models with constant demand. Sarkar et al. (1997) developed inventory model with dependent demand and deterioration. Liao et al. (2000) developed an economic quantity model for deteriorating items and considered delay in payment under inflation. Ouyang et al. (2006) investigated a inventory model for non-instantaneous deteriorating products with permissible delay in payments. In order to account for partial backorders, Hung (2011) changed the inventory model's ramp type demand rate and Weibull degradation rate to arbitrary demand rate and arbitrary deterioration rate. Lee et al. (2012) created an economic quality order for deteriorating products under stock-dependent demand with controllable deteriorating. Sarkar and sarkar (2013) examined inventory model with variable deterioration and demand. Jaggi et al. (2015) developed inventory model for two-warehouses with effect of deterioration. Sangal et al. (2016) developed a inventory model with partial backlogging and learning effect under fuzzy environment. Jaggi et al. (2017) developed two-warehouses model with permissible delay in payment for deteriorating item. Mahapatra et al. (2017) developed inventory model for deteriorating product with reliability dependent demand under learning effect. Sharma et al. (2019) created economic order quantity model with price and time dependent demand and also consider deteriorating product. Xu et al. (2021) developed model for two ware-houses under trapezoidal-type demand and learning effect. Kumar et al. (2021) developed inventory model for smart items and two warehouses with carbon emissions. Das (2023) developed two ware houses production inventory model and use a very useful algorithm that is genetic algorithm. Chandramohan et al. (2023) developed a inventory model for non-instantaneous deteriorating items in supply chain. From this existing literature there is no one model in which non-instantaneous item with two-warehouses, learning effect in supply chain.

II. NOTATIONS AND ASSUMPTIONS

The following assumptions are considering for this model:

1. Demand considering depend on time and selling price $D(p,t) = a - bp + ct, a > b, a > c$.
2. Length of replenishment cycle is constant and unknown.
3. Own warehouses have fixed quantity and rented warehouses have unlimited capacity.

4. Shortages are allowed with partial backlogged.
5. The partial backlogging rate for negative inventory is $B(t) = \frac{1}{e^{\delta(T-t)}} : \delta > 0$.
6. Inflation is also considered.
7. Effect of learning on ordering and holding costs are taken into account.

Parameters	
a	Constant
b	Constant
p	Selling price
c	Constant
W	Storage capacity of OW
R	Inventory level
d	Purchase cost
$I_{r_1}(t)$	Inventory level of RW at $0 \le t \le t_1$
$I_{r_2}(t)$	Inventory level of RW at $t_1 \le t \le t_2$
$I_{r_3}(t)$	Inventory level of RW at $t_2 \le t \le t_3$
$I_{w_1}(t)$	Inventory level of OW at $0 \le t \le t_1$
$I_{w_2}(t)$	Inventory level of OW at $t_1 \le t \le t_2$
$I_{w_3}(t)$	Inventory level of OW at $t_2 \le t \le t_3$
$I_{w_4}(t)$	Inventory level of OW at $t_3 \le t \le t_4$
$I_s(t)$	Backlogged level at time t
δ	Backlogging parameter
S	Amount of backlogged demand
A	Ordering cost
A'	Ordering cost due to carbon emission
m	Inflation rate
H_r	Holding cost for rented warehouse
H'	Holding cost for rented warehouse due to carbon emission
H_o	Holding cost for own warehouses
H''	Holding cost for own warehouses due to carbon emission
C_b	Shortage cost
C_o	Lost sale cost
n	Cumulative frequency
σ	Learning rate
A''	Constant
H'''	Constant
Decision variables	
t_3	Time where inventory level is zero in RW
T	Length of the replenishment cycle

III. MATHEMATICAL MODELLING

The inventory system for rapidly degrading items with shortages is illustrated. Initially, lot size Q enters the system; after satisfying the backorder from the previous cycle, the remaining initial inventory is R. Out of these R units, W units are stored in the own warehouse, while the remaining (R - W) units are housed in the rented warehouse. First, the merchandise from the rented warehouse is utilized. There is no decay in the time interval [0, 1]. Inventory in the rated warehouse drops after time t_1 due to demand (D). The remaining inventory in the rented warehouse is $R - W - r_1$. During the next time interval $[t_1, t_2]$, the rented warehouse experiences linear decay and demand, resulting in a reduction in inventory r_2. After time t_2, the remaining inventory in the rented storage is $R - W - r_1 - r_2$ .The nonlinear decline begins only after time t_2. The rented warehouse's stock is exhausted due to nonlinear deterioration and demand at time t_3. The inventory stored in the company's warehouse is now being used. Currently, the own warehouse holds inventory $W - w_1 - w_2$, while the need during time interval $[0, t_3]$ is met by restocking from the rented warehouse. Linear decay in time interval $[t_1, t_2]$ depletes w_1 inventory, while non-linear decay in time interval $[t_2, t_3]$ reduces w_2 inventory in the warehouse. After time $t3$, the own warehouse holds inventory $W - w_1 - w_2$, leaving the rented warehouse empty. During the time interval$[t_3, t_4]$, due to demand and decay, the warehouse becomes empty. The demand is partially backlogged throughout the time interval $[t_4, T]$. The inventory levels are determined by the differential equation:

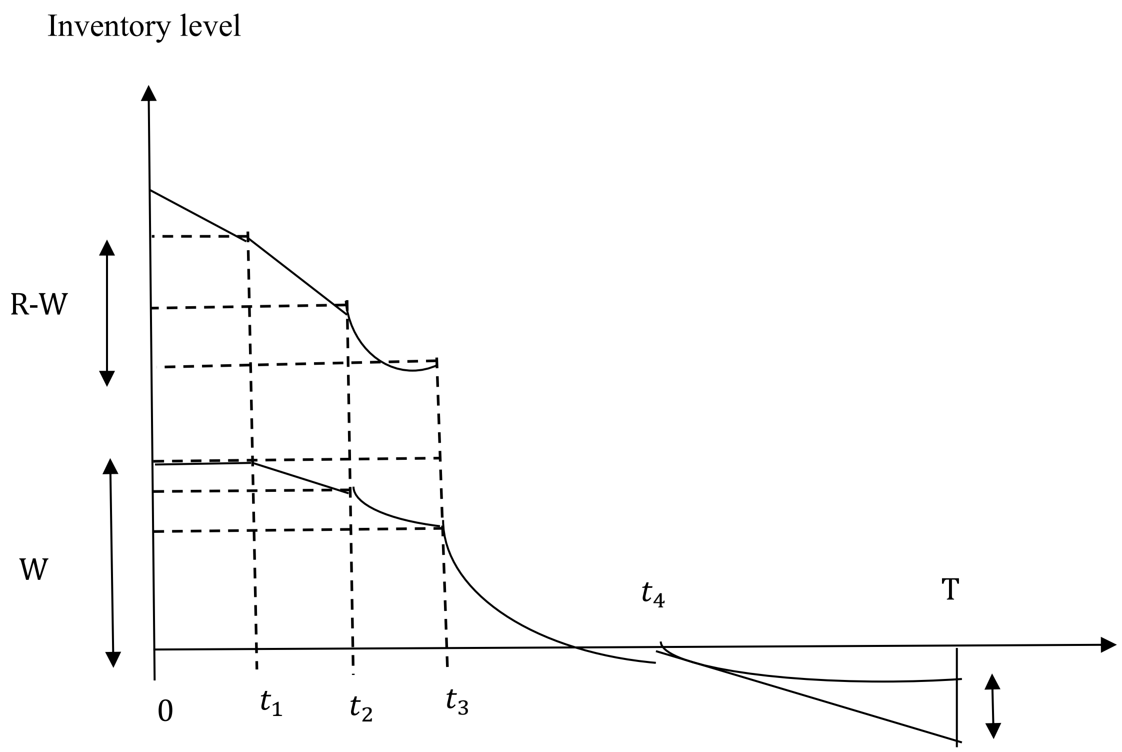


Fig:1-Inventory level of this model

$$\frac{dI_{r_1}(t)}{dt} = -(D + ct): 0 \le t \le t_1 \quad (1)$$

$$with\ BC: I_{r_1}(0) = R - W, I_{r_1}(t_1) = R - W - r_1 \quad (2)$$

$$\frac{dI_{r_2}(t)}{dt} + \theta_l(t)I_{r_2}(t) = -(D + ct): t_1 \le t \le t_2 \quad (3)$$

$$with\ BC: I_{r_2}(t_1) = R - W - r_1, I_{r_2}(t_2) = R - W - (r_1 - r_2) \quad (4)$$

$$\frac{dI_{r_3}(t)}{dt} + \theta_n(t)I_{r_3}(t) = -(D + ct): t_2 \le t \le t_3 \quad (5)$$

$$with\ BC: I_{r_3}(t_2) = R - W - (r_1 + r_2), I_{r_3}(t_3) = 0 \quad (6)$$

$$\frac{dI_{w_1}(t)}{dt} = 0: 0 \le t \le t_1 \quad (7)$$

$$with\ BC: I_{w_1}(0) = W, I_{w_1}(t_1) = W \quad (8)$$

$$\frac{dI_{w_2}(t)}{dt} + \theta_l(t)I_{w_2}(t) = 0: t_1 \le t \le t_2 \quad (9)$$

$$with\ BC: I_{w_2}(t_1) = W, I_{w_2}(t_2) = W - w_1 \quad (10)$$

$$\frac{dI_{w_3}(t)}{dt} + \theta_n(t)I_{w_3}(t) = 0: t_2 \le t \le t_3 \quad (11)$$

$$with\ BC: I_{w_3}(t_2) = W - w_1, I_{w_3}(t_3) = W - (w_1 + w_2) \quad (12)$$

$$\frac{dI_{w_4}(t)}{dt} + \theta_n(t)I_{w_4}(t) = -(D+ct): t_3 \le t \le t_4 \quad (13)$$

$$with\ BC: I_{w_4}(t_3) = W - (w_1 - w_2), I_{w_4}(t_4) = 0 \quad (14)$$

$$\frac{dI_s(t)}{dt} = \frac{D+ct}{e^{\delta(T-t)}}: t_4 \le t \le T \quad (15)$$

$$with\ BC: I_s(t_4) = 0, I_s(T) = S \quad (16)$$

The solution to Eq. (1), with specified boundary condition, is

$$I_{r_1}(t) = -Dt - \frac{ct^2}{2} + R - W \quad (17)$$

$$r_1 = Dt_1 + \frac{ct_1^2}{2} \quad (18)$$

The solution to Eq. (2), with specified boundary condition, is

$$I_{r_2}(t) = (R - W - r_1)e^{X_l(t_1)-X_l(t_2)} + e^{-X_l(t_2)}\int_{t_2}^{t_1} e^{X_l(v)}(D+cv)dv \quad (19)$$

$$r_2 = (R - W - r_1)\left(1 - e^{X_l(t_1)-X_l(t_2)}\right) - e^{-X_l(t_2)}\int_{t_2}^{t_1} e_l^{X_l(v)}\,(D+cv)dv \quad (20)$$

$$r_2 = \left(R - W - Dt_1 - \frac{ct_1^2}{2}\right)\left(1 - e^{X_l(t_1)-X_l(t_2)}\right) - e^{-X_l(t_2)}\int_{t_2}^{t_1} e_l^{X_l(v)}\,(D+cv)dv \quad (21)$$

Where $X_l(t) = \int \theta_l(v)dv$

The solution to Eq. (3), with specified boundary condition, is

$$I_{r_3}(t) = e^{-X_n(t)}\int_t^{t_3}(D+cv)\,e^{X_n(t)}dv \quad (22)$$

$$R = W + r_1 + r_2 + e^{-X_n(t_2)}\int_{t_2}^{t_3}(D+cv)\,e^{X_n(v)}dv \quad (23)$$

$$where\ X_n(t) = \int_0^t \theta_n(v)\,dv$$

The solution to Eq. (4), with specified boundary condition, is

$$I_{w_1}(t) = W \quad (24)$$

The solution to Eq. (5), with specified boundary condition, is

$$I_{w_2}(t) = W e^{X_l(t_1)-X_l(t)} \quad (25)$$

$$w_1 = W\left(1 - e^{X_l(t_1)-X_l(t_2)}\right) \quad (26)$$

$$X_l(t) = \int \theta_l(v)dv$$

The solution to Eq. (6), with specified boundary condition, is

$$I_{w_3}(t) = (W - w_1)e^{X_n(t_2)-X_n(t)} \quad (27)$$

$$w_2 = (W - w_1)\left(1 - e^{X_n(t_2)-X_n(t_3)}\right) \quad (28)$$

$$where\ X_n(t) = \int_0^t \theta_n(v)\,dv$$

The solution to Eq. (7), with specified boundary condition, is

$I_{w_4}(t) = e^{-(X_n(t))} \int_t^{t_4} e^{X_n(v)} (D + cv)dv$ (29)

$W = w_1 + w_2 + e^{-X_n(t_3)} \int_{t_3}^{t_4} e^{X_n(v)} (D + cv)dv$ (30)

$$where\ X_n(t) = \int_0^t \theta_n(v)\, dv$$

The solution to Eq. (8), with specified boundary condition, is

$I_s(t) = \left[e^{-(T-t)\delta} - e^{-(T-t_4)\delta}\right]\left[\frac{D}{\delta} + \frac{c}{\delta} - \frac{c}{\delta^2}\right]$ (31)

$S = \left[1 - e^{-(T-t_4)\delta}\right]\left[\frac{D}{\delta} + \frac{c}{\delta} - \frac{c}{\delta^2}\right]$ (32)

Now by using above equations, we get the maximum positive inventory-

$R = W + Dt_1 + \frac{ct_1^2}{2} + \left(R - W - Dt_1 + \frac{ct_1^2}{2}\right)\left(1 - e^{X_l(t_1) - X_l(t)}\right) - e^{-X_l(v)} \int_t^{t_1} e_l^{X_l(v)} (D + cv)dv + e^{-X_n(t_2)} \int_{t_2}^{t_3} (D + cv)\, e^{X_n(v)} dv$ (33)

$R = W + Dt_1 + \frac{ct_1^2}{2} - e^{-X_l(t_2)} \int_{t_2}^{t_1} e_l^{X_l(v)} (D + cv)dv + e^{X_l(t_2) - X_l(t_1) - X_n(t_2)} \int_{t_2}^{t_3} (D + cv)\, e^{X_n(v)} dv$ (34)

Where $W = e^{X_l(t_2) - X_l(t_1) - X_n(t_2)} \int_{t_3}^{t_4} e^{X_n(v)} (D + cv)dv$ (35)

Thus, the order quantity for the replenishment cycle is obtained as:

$Q = R + S = W + Dt_1 + \frac{ct_1^2}{2} - e^{-X_l(t_1)} \int_{t_2}^{t_1} e_l^{X_l(v)} (D + cv)dv + e^{X_l(t_2) - X_l(t_1) - X_n(t_2)} \int_{t_2}^{t_3} (D + cv)\, e^{X_n(v)} dv + \left[1 - e^{-(T-t_4)\delta}\right]\left[\frac{D}{\delta} + \frac{c}{\delta} - \frac{c}{\delta^2}\right]$ (36)

$Q = Dt_1 + \frac{ct_1^2}{2} - e^{-X_l(t_1)} \int_{t_2}^{t_1} e^{X_l(v)} (D + cv)dv + e^{X_l(t_2) - X_l(t_1) - X_n(t_2)} \int_{t_2}^{t_4} (D + cv)\, dv + \left[1 - e^{-(T-t_4)\delta}\right]\left[\frac{D}{\delta} + \frac{c}{\delta} - \frac{c}{\delta^2}\right]$ (37)

Retailer's cost: The total cost (TC) per cycle encompasses the following components:

i. The ordering cost per cycle:

$OC = \frac{A + A'}{T} + \frac{A''}{n^\sigma}$ (38)

ii. Holding cost per cycle in RW:

$$HC_{rw} = \frac{H_r + H'}{T}\left\{\int_0^{t_1} e^{-mt}\, I_{r_1}(t)\, dt + \int_{t_1}^{t_2} e^{-mt}\, I_{r_2}(t)\, dt + \int_{t_2}^{t_3} e^{-mt}\, I_{r_3}(t)\, dt\right\}$$

$= \frac{H_r + H'}{T}\left\{\int_0^{t_1} e^{-mt}\left(-Dt - \frac{ct^2}{2} + R - W\right) dt + \int_{t_1}^{t_2} e^{-mt}\left((R - W - r_1) e^{X_l(t_1) - X_l(t)} + e^{-X_l(v)} \int_t^{t_1} e^{X_l(v)} (D + cv)dv\right) dt + \int_{t_2}^{t_3} e^{-mt}\left(e^{-X_n(t)} \int_t^{t_3} (D + cv)\, e^{X_n(t)} dv\right)\ dt\right\}$ (39)

iii. Holding cost per cycle in OW:

$HC_{ow} = \frac{H_0 + H'{\prime}}{T} + \frac{H'''}{n^\sigma}\left\{\int_0^{t_1} e^{-mt} I_{w_1}(t)\, dt + \int_{t_1}^{t_2} e^{-mt} I_{w_2}(t)\, dt + \int_{t_2}^{t_3} e^{-mt} I_{w_3}(t)\, dt + \int_{t_3}^{t_4} e^{-mt}\, I_{w_4}(t)\, dt\right\}$

$= \frac{H_0 + H''}{T}\left\{\int_0^{t_1} e^{-mt} W\, dt + \int_{t_1}^{t_2} \left(W e^{X_l(t_1) - X_l(t)}\right) e^{-mt}\, dt + \int_{t_2}^{t_3} e^{-mt} (W - w_1) e^{X_n(t_2) - X_n(t)}\, dt + \int_{t_3}^{t_4} \left(e^{-(X_n(t))} \int_t^{t_4} e^{X_n(v)} (D + cv)dv\right) e^{-mt}\, dt\right\}$ (40)

iv. Shortage cost:

$$SC = \frac{C_b}{T}\int_{t_4}^{T} e^{-mt} I_s(t)\, dt$$

$$= \frac{C_b}{T}\int_{t_4}^{T} e^{-mt}\left[e^{-(T-t)\delta} - e^{-(T-t_4)\delta}\right]\left[\frac{D}{\delta} + \frac{c}{\delta} - \frac{c}{\delta^2}\right] dt$$

$$= \frac{C_b}{T}\left(\frac{D}{\delta} + \frac{c}{\delta} - \frac{c}{\delta^2}\right)\left[\frac{1}{\delta - m}\left(e^{-mT} - e^{t_4(\delta-m)-T\delta}\right) + \frac{1}{m}\left(e^{-mT+\delta(t_4-T)} - e^{-mt_4+\delta(t_4-T)}\right)\right] \quad (41)$$

v. Lost sales cost:

$$LSC = \frac{C_0}{T}\int_{t_4}^{T} (D + ct)\left(1 - e^{-\delta(T-t)}\right)e^{-mt} dt$$

$$= \frac{C_0}{T}\left[\frac{D}{m}\left(e^{-mt_4} - e^{-mT}\right) + \frac{D}{\delta-m}\left(e^{-\delta T+(\delta-m)t_4} - e^{-mT}\right) - \frac{c}{m}\left(Te^{-mT} + t_4e^{-mt_4}\right) - c\left(\frac{e^{-mT}}{T^2} + \frac{e^{-mt_4}}{t_4^2}\right) - \frac{c}{\delta-m}\left(Te^{-mT} + t_4e^{-\delta T+(\delta-m)t_4}\right) - \frac{c}{(\delta-m)^2}\left(e^{-mT} + e^{-\delta T+(\delta-m)t_4}\right)\right] \quad (42)$$

vi. Purchasing cost:

$$PC = \frac{dQ}{T}$$

$$= \frac{d}{T}\left[Dt_1 + \frac{ct_1^2}{2} - e^{-X_l(t_1)}\int_{t_2}^{t_1} e^{X_l(v)}(D + cv)dv + e^{X_l(t_2)-X_l(t_1)-X_n(t_2)}\int_{t_2}^{t_4}(D + cv)\, dv + \left[1 - e^{-(T-t_4)\delta}\right]\left[\frac{D}{\delta} + \frac{c}{\delta} - \frac{c}{\delta^2}\right]\right]$$
(43)

$$Total\ average\ cost = \frac{1}{T}\{OC + HC_{rw} + HC_{ow} + SC + LSC + PC\}$$

$$TAC = \Bigg\{\frac{A+A'}{T} + \frac{H_r+H'}{T}\Big\{\int_0^{t_1} e^{-mt}\left(-Dt - \frac{ct^2}{2} + R - W\right)dt + \int_{t_1}^{t_2} e^{-mt}\left((R - W - r_1)e^{X_l(t_1)-X_l(t)} + e^{-X_l(v)}\int_t^{t_1} e^{X_l(v)}(D + cv)dv\right)dt + \int_{t_2}^{t_3} e^{-mt}\left(e^{-X_n(t)}\int_t^{t_3}(D + cv)\, e^{X_n(t)}dv\right)\, dt\Big\} + \frac{H_o+H''}{T}\Big\{\int_0^{t_1} e^{-mt}W\, dt + \int_{t_1}^{t_2}\left(We^{X_l(t_1)-X_l(t)}\right)e^{-mt}\, dt + \int_{t_2}^{t_3} e^{-mt}(W - w_1)e^{X_n(t_2)-X_n(t)}\, dt + \int_{t_3}^{t_4}\left(e^{-(X_n(t))}\int_t^{t_4} e^{X_n(v)}(D + cv)dv\right)e^{-mt}\, dt\Big\} + \frac{C_b}{T}\left(\frac{D}{\delta} + \frac{c}{\delta} - \frac{c}{\delta^2}\right)\left[\frac{1}{\delta-m}\left(e^{-mT} - e^{t_4(\delta-m)-T\delta}\right) + \frac{1}{m}\left(e^{-mT+\delta(t_4-T)} - e^{-mt_4+\delta(t_4-T)}\right)\right] + \frac{C_0}{T}\left[\frac{D}{m}\left(e^{-mt_4} - e^{-mT}\right) + \frac{D}{\delta-m}\left(e^{-\delta T+(\delta-m)t_4} - e^{-mT}\right) - \frac{c}{m}\left(Te^{-mT} + t_4e^{-mt_4}\right) - c\left(\frac{e^{-mT}}{T^2} + \frac{e^{-mt_4}}{t_4^2}\right) - \frac{c}{\delta-m}\left(Te^{-mT} + t_4e^{-\delta T+(\delta-m)t_4}\right) - \frac{c}{(\delta-m)^2}\left(e^{-mT} + e^{-\delta T+(\delta-m)t_4}\right)\right] + \frac{d}{T}\left[Dt_1 + \frac{ct_1^2}{2} - e^{-X_l(t_1)}\int_{t_2}^{t_1} e^{X_l(v)}(D + cv)dv + e^{X_l(t_2)-X_l(t_1)-X_n(t_2)}\int_{t_2}^{t_4}(D + cv)\, dv + \left[1 - e^{-(T-t_4)\delta}\right]\left[\frac{D}{\delta} + \frac{c}{\delta} - \frac{c}{\delta^2}\right]\right]\Bigg\}$$
(44)

Our purpose is to minimize the total average cost per cycle.

IV. SOLUTION METHODOLOGY

For optimal values of t_3 and T that minimize total cost following steps are taken:

1. Differentiating equation (44) with respect tot_3 and T.
2. Put these equations equal to zero, we have

$$\frac{d(TC)}{dt_3} = 0, \frac{d(TC)}{dT} = 0$$

And find critical points.

3. Find second order derivative of equation (44) and check the condition
$$\frac{d^2(TC)}{dt_3^2} > 0, \frac{d^2(TC)}{dT^2} > 0 \; and \; \frac{d^2(TC)}{dt_3^2} * \frac{d^2(TC)}{dT^2} - \frac{d^2(TC)}{dt_3 dT} > 0$$
For the minimum cost.

V. NUMERICAL ANALYSIS

We know that deterioration is not always constant, there are different situation for different items so for numerical validation we use different realistic situation of deterioration, here we show two cases:

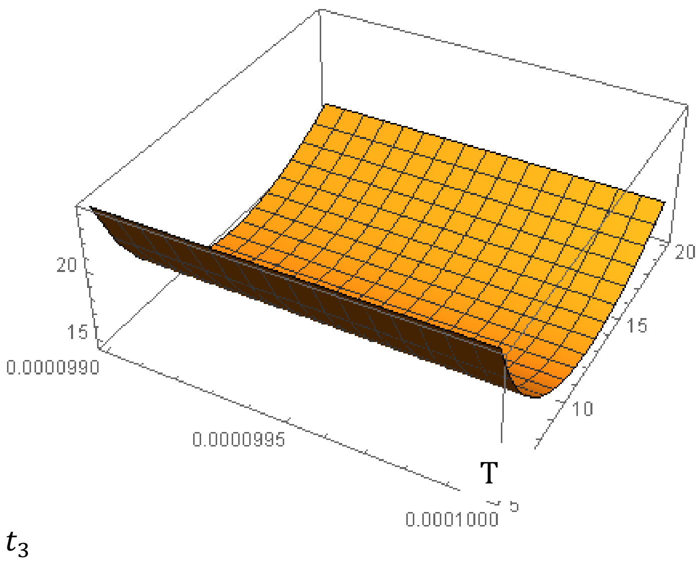
For case 1 take $\theta_l(t) = \alpha + \beta t$ and $\theta_n(t) = \frac{1}{\mu - \tau t}$

Parameters	Values	Parameters	Values
A	1000	a	100
A'	100	b	0.01
H_r	45	p	39
H'	0.2	c	0.02
H_o	400	δ	0.005
$H"$	30	m	0.5
C_b	0.78	C_o	200
d	42	μ	0.05
α	0.01	τ	0.001
β	0.003	t_1	0.00002
t_2	0.000052	t_4	14.08
n	0.07	A'''	5.5
σ	0.64	H'''	2.45

Optimal Solution

Parameters	Values
t_3	0.0000995
T	15.0008
$Total\ cost$	20.4367

Graphical representation of this optimal solution is shown below:



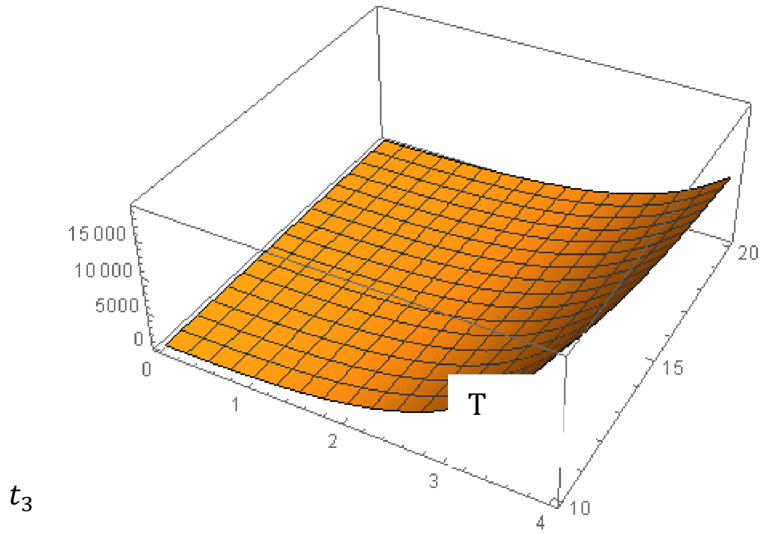
For case 2: take $\theta_l(t) = \alpha + \beta t + \gamma t^2$ and $\theta_n(t) = \frac{1}{\mu - \tau t - \omega t^2}$

Parameters	Values	Parameters	Values
A	100	a	10000
A'	70	b	20
H_r	36	p	10
H'	0.02	c	0.02
H_o	0.5	δ	0.003
H''	12	m	0.7
C_b	700	C_o	20
d	20	μ	0.02
α	0.03	τ	0.001
β	0.001	t_1	1.2
t_2	2.5	t_4	3.903
γ	0.002	ω	0.0001
n	0.03	A'''	0.55
σ	0.42	H'''	0.5

Optimal solution

Parameters	Values
t_3	3.2467
T	4.589251
$Total\ cost$	5030.982

Graphical representation is



VI. SENSITIVITY ANALYSIS

For case 1:

Parameters	% change	t_3	T	Total cost
A	+20%	0.0000425	15.5708	23.4367
	+10%	0.00005241436	16.9827	20.874
	-10%	0.00008376	17.364	32.475
	-20%	0.00005267	18.876	-
a	+20%	0.0000636	16.98	-
	+10%	0.00006286	16.98	16.374
	-10%	0.00006564	16.98	19.4898
	-20%	0.0000657	16.98	26.1582
b	+20%	0.00005424	13.82763	34.651
	+10%	0.00001581	25.7365	12.325

	-10%	0.00001865	14.3654	27.258
	-20%	0.00002548	15.3754	-
p	+20%	0.000057	16.3754	16.374
	+10%	0.0000351	19.365	19.4898
	-10%	0.0000675	12.765	26.1582
	-20%	0.0000841	11.435	34.651
A'	+20%	0.00009514	11.365	12.325
	+10%	0.00002152	21.4562	27.258
	-10%	0.0000484	11.364	25.125
	-20%	0.00002671	21.36445	-
c	+20%	0.0000685	20.46	25.126
	+10%	0.00006821	13.82763	43.279
	-10%	0.0000294	25.7365	35.451
	-20%	0.000026781	14.3654	35.451
H_r	+20%	0.00004813	15.3754	30.254
	+10%	0.000063	16.3754	30.254
	-10%	0.00005411	19.365	30.454
	-20%	0.00005541	12.765	-
H'	+20%	0.0000841	11.435	52.365
	+10%	0.00009514	11.365	50.125
	-10%	0.00002152	21.4562	25.126
	-20%	0.0000484	11.364	43.279
H_o	+20%	0.00002671	21.36445	35.451
	+10%	0.0000685	20.46	35.451
	-10%	0.00006821	13.82763	30.254
	-20%	0.0000294	25.7365	30.254
m	+20%	0.0000636	14.3654	35.654
	+10%	0.00006286	15.3754	951.24
	-10%	0.00006564	16.3754	52.369
	-20%	0.0000657	19.365	-
C_b	+20%	0.00005424	12.765	2587.32
	+10%	0.00001581	11.435	258.21
	-10%	0.00001865	11.365	5236.21
	-20%	0.00002548	21.4562	-
C_o	+20%	0.000057	11.364	25.126
	+10%	0.0000636	21.36445	43.279
	-10%	0.00006286	20.46	35.451
	-20%	0.00006564	-	35.451
δ	+20%	0.0000657	11.234	30.254
	+10%	0.00005424	11.456	30.254
	-10%	0.00001581	22.987	27.125
	-20%	0.00001865	-	-
d	+20%	0.00002548	-	54.215
	+10%	0.000057	-	24.325
	-10%	0.0000457	-	24.325
	-20%	0.0000575	31.487	24.354

For case 2:

a	+20%	3.2467	4.589251	5030.982
	+10%	3.0245	4.68298	4356.345
	-10%	3.8467	2.4622	2425.836
	-20%	2.3456	2.01872	2354.24
b	+20%	1.3475	1.9343	-
	+10%	3.8765	3.9265	-
	-10%	4.6743	6.8267	-
	-20%	5.342	7.4973	-
c	+20%	4.7642	5.3671	3455.987
	+10%	-	4.86627	5342.875
	-10%	-	7.35246	3547.76

	-20%	-	3.7267	2346.376
p	+20%	3.2467	4.589251	5030.982
	+10%	3.0245	4.68298	4356.345
	-10%	3.8467	2.4622	2425.836
	-20%	2.3456	2.01872	2354.24
A	+20%	-	-	-
	+10%	1.3476	8.256	5098.76
	-10%	3.987	7.3643	4555.92
	-20%	-	-	-
A'	+20%	3.2467	4.589251	5030.982
	+10%	3.0245	4.68298	4356.345
	-10%	3.8467	2.4622	2425.836
	-20%	2.3456	2.01872	2354.24
H_r	+20%	1.2346	5.3671	3455.987
	+10%	1.6543	4.86627	5342.875
	-10%	1.90398	7.35246	3547.76
	-20%	2.476	3.7267	2346.376
H'	+20%	2.04986	4.375	3547.76
	+10%	2.347	4.6853	2346.376
	-10%	3.9583	5.356	6547.54
	-20%	3.0126	6.439	6547.98
δ	+20%	3.2467	4.589251	6583.87
	+10%	3.0245	4.68298	5639.98
	-10%	3.8467	2.4622	4567.837
	-20%	2.3456	2.01872	3422.678
H''	+20%	4.589251	5.3671	3455.987
	+10%	4.68298	4.86627	5342.875
	-10%	2.4622	7.35246	3547.76
	-20%	2.01872	3.7267	2346.376
C_b	+20%	3.2467	4.589251	5030.982
	+10%	3.0245	4.68298	4356.345
	-10%	3.8467	2.4622	2425.836
	-20%	2.3456	2.01872	2354.24
C_o	+20%	3.9832	5.3671	3455.987
	+10%	3.9832	4.86627	5342.875
	-10%	3.9832	7.35246	3547.76
	-20%	3.9832	3.7267	2346.376
d	+20%	4.9836	4.9976	4356.345
	+10%	5.234	5.73930	2425.836
	-10%	8.5764	9.4753	2354.24
	-20%	2.334	7.5342	3455.987
m	+20%	4.2536	7.35246	5342.875
	+10%	5.2847	3.7267	3547.76
	-10%	3.8476	4.9976	2346.376
	-20%	2.345	5.73930	6547.54
H_o	+20%	4.9836	9.4753	6547.98
	+10%	5.234	7.5342	6583.87
	-10%	8.5764	9.6353	5639.98
	-20%	2.334	5.435	4567.837

VII. RESULT AND DISCUSSION

For case 1, On changing parameter A, t_3, T and Total cost are all fluctuating. For case 2, on changing parameter A, t_3 is decreasing, T and total cost are increasing. For case 1, on changing parameter a, t_3 is increasing, T is constant and total cost is decreasing. For case 2, on changing parameter a, t_3 is fluctuating, T is increasing and total cost is increasing. For case 1, On changing parameter c, t_3, T and Total cost are all fluctuating. For case 2, on changing parameter c, t_3 is decreasing, T and total cost are increasing. For case 1, on changing parameter p, t_3 is increasing, T is constant and total cost is decreasing. For case 2, on changing parameter p, t_3 is fluctuating, T is increasing and total cost is increasing. For both cases on changing C_b, t_3, T and total cost all are increasing. For both cases on changing d, t_3, T and total cost all are decreasing. On changing parameter H_o, t_3, T and Total cost are all increasing. For case 2, on changing parameter H_o, t_3 is fluctuating, T and total cost are increasing. On changing parameter C_o, t_3, T and Total cost are all increasing. For case 2, on changing parameter C_o, t_3 is constant, T and total cost are increasing.

VIII. CONCLUSION

this study has shed light on the multifaceted dynamics of inventory management within dual warehouse systems, emphasizing the significance of considering diverse factors such as linear and nonlinear deterioration rates, carbon emissions, inflation, and dynamic demand dependencies. By integrating theoretical insights with empirical analysis, we have deepened our understanding of how these factors interact and influence inventory control strategies. The findings underscore the importance of adopting adaptive approaches that balance operational efficiency with environmental sustainability and economic viability. As organizations strive to navigate an increasingly complex business landscape, the insights provided by this research offer actionable recommendations for optimizing inventory management practices. This study can be extended with more ware-houses, and stock depended demand also, this model can be extended with fuzzy and variable holding cost. By embracing innovation and leveraging advanced analytical tools, businesses can enhance their resilience, responsiveness, and competitiveness in the face of evolving challenges. Moving forward, continued research and collaboration will be essential to further refine inventory management strategies and drive sustainable growth in dual warehouse inventory systems.

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