Pulsatile Bingham Plastic Fluid Flow through an Inclined Asymmetric Stenosed Artery

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Abstract— The cardiovascular disease patients are more at risk of having stenosis disease due to blocked and narrowed blood arteries. The need of creating a mathematical model to recognize and understand the hemodynamic characteristics of blood rheology cannot be overstated. The current work uses erratic blood flow through an inclined stenosed artery and treats blood as a non-Newtonian fluid, such as Bingham Plastic fluid. The Perturbation approach has been used to solve the non-linear differential equations of the aforementioned mathematical model combined with the suitable boundary conditions guiding the fluid flow. With the aid of MATLAB, positive numerical computation findings have been determined along with the body acceleration, slip condition, and asymmetric stenosis effect at the stenosed wall. For the Bingham Plastic fluid, the analysis of flow parameters such as plug core velocity, plug core radius, wall shear stress, volumetric flow rate, and effective viscosity has been optimized under a variety of rheological factors including yield stress, inclination, the size of the stenosis's effect, slip, and body acceleration.

Keywords: Bingham Plastic fluid, Inclination, Plug core region, Stenosis, Unsteady flow.

I. INTRODUCTION

The circulatory flow studies in blood arteries are receiving a lot of interest right now since they account for roughly 75% to 80% of mortality. Cardiovascular disease known as atherosclerosis (or stenosis) causes arteries to narrow, causing a circulation condition. According to Dwyer et al. [4], intravascular plaque and other waste products that have formed inside of and on the walls of blood vessels impede blood flow. The interesting finding that Bingham plastic fluid provides more precise information on the rheological characteristics of blood was made by Eirich [5] and Fung [8]. The oscillating stream of a Bingham plastic in rigid and intermittently uprooted tubes has been the subject of an annoyance show by Dechant [3]. Due to the non-Newtonian nature of blood, Bingham's plastic liquid and experiments showed that stream and divider shear push resistance increase with the degree of stenosis, according to Verma [18].

Chien [2] focused on his research on diseases such myocardial infarction, hypertension, angina, silent ischemia, etc. and showed that blood in a diseased state has non-Newtonian properties. According to Chaturani et al. [1], for a tube with a diameter of 0.095mm, the Herschel Bulkley fluid represents blood more accurately than the Power law and Bingham fluids. A theoretical investigation of Herschel Bulkley flow across a stenosed artery section was presented by Mishra and Shit [10]. According to Blair and Spanner [14], blood can be more accurately represented by Casson fluid in the medium range of shear rate since their representations of fluids are similar. In their comparison of the two-fluid Casson and Herschel Bulkley models, Sankar and Ismail [12] found that the two-fluid Casson model is a superior description of blood in a sick state. Young and Tsai [19] studied the impact of axisymmetric and non-symmetric stenosis on the blood flow via a tube. Through a stenosed tube, Tu and Deville [17] explained and contrasted the Bingham plastic, Power-law, and Herschel-Bulkley fluid flows. In order to simulate the flow of fluid via stenosed arteries under the influence of body acceleration, Siddiqui et al. [16] and Sankar and Lee [13] devised a non-Newtonian pulsatile fluid flow model. El-Shahed [6] followed El-Shawey et al. [7] in extending their description of the body acceleration impact on the pulsatile flow of blood to a stenosed porous medium. With periodic body acceleration through an elastic stenosed artery, Mandal et al. [9] developed a two-dimensional non-Newtonian Power-law model. Fluid velocity, flow rate, wall shear stress, and resistance are all significantly impacted by body acceleration and yield stress in the occluded artery, according to Nagarani and Sarojamma's [11] observations.

A non-Newtonian fluid flow model with slip condition on blood flow through a stenosed artery using power-law fluid examined by Shrivastava et al. [20]. The effect of stenotic geometry and non-Newtonian property of blood flow through arterial stenosis investigated by Sriyab [21]. Kumar et al. [22] described the comparative study of non-Newtonian physiological blood flow through elastic stenotic pipe. Dhange et al. [23] discussed blood flow via an inclined tube under the influence of magnetic field and a constant incompressible Casson fluid.

In this investigation, the non-Newtonian pulsatile blood flow through an inclined stenosed artery is studied. Bingham Plastic, a non-Newtonian fluid, has been used here. In the future, we can predict how Newtonian fluid and other non-Newtonian fluid and presence of nanoparticles will affect this model's behaviour.

2. MATHEMATICAL FORMULATION

Let us consider an axially symmetric, fully developed, laminar, and unsteady (pulsatile) blood flow in the axial direction z' in an inclined arterial segment with stenosis in the lumen, under the effect of slip and body acceleration. It is also assumed that the presence of a pressure gradient generates the unsteady nature in the flow of blood in the stenosed artery. Let (r', ϕ', z') be the cylindrical polar co-ordinates, where r' denotes the radial co-ordinate while ϕ' and z' denotes the azimuthal angle and axial co-ordinate respectively.

The radius of the artery can be mathematically expressed as (see Fig.1):

$$\left.\begin{aligned} \frac{R'(z')}{R'_0} &= 1-\eta'\left[L_s'^{(h-1)}(z'-d')-(z'-d')^h\right], \quad d' \le z' \le d'+L'_s \\ &= 1, \qquad\qquad \text{otherwise} \end{aligned}\right\} \tag{1}$$

Where $R'(z')$ and R'_0 is the radius of the artery with and without stenosis respectively, d' is the location of the stenosis, L'_s is the length of the stenosis and $h \ge 2$ is the stenosis shape parameter and stenosis is radially symmetric at h = 2. Where the parameter R'_0 is defined as

$$\eta' = \frac{\delta'}{R'_0 L'^{h}_{s}} \frac{h^{h/(h-1)}}{(h-1)}$$

Where δ' denotes the maximum height of the stenosis at $z' = d' + L'_s / h^{1/(h-1)}$ such that $\delta' / R'_0 << 1$.

The momentum equations governing the fluid flow are

$$-\left(\frac{\partial p'}{\partial z'}\right)-\left(\frac{1}{r'}\right)\frac{\partial}{\partial r'}\left(r'\tau'_s\right)+B'_a(t') = \rho'\left\{\left(\frac{\partial v'_a}{\partial t'}\right)-g\sin\gamma\right\} \tag{2}$$

$$\frac{\partial p'}{\partial r'} = 0 \tag{3}$$

Where v'_a denotes the axial velocity along z' direction, p' denotes the pressure, t' denotes the time, ρ' denotes the density, τ'_s denotes the wall shear stress, γ denotes the inclination angle, g denotes the acceleration due to gravity and $B'_a(t')$ denotes the periodic body acceleration in the axial direction, mathematically equation (4) defined this body acceleration.

$$B'_a(t') = a_0 \cos\left(\omega'_{ba} t' + \phi\right) \tag{4}$$

Where a_0 and ϕ denotes the amplitude, phase angle of body acceleration respectively with respect to the pressure gradient. $\omega'_{ba} = 2\pi f'_b$ here f'_b denotes the frequency (in Hz). Also wave effect can be neglected by assuming the frequency of the body acceleration f'_b to be small.

Now the pressure gradient can be defined as

$$\frac{-\partial p'}{\partial z'}\left(z',t'\right) = A_0 + A_1\cos\left(\omega'_{pa}t'\right), \quad t' \ge 0 \tag{5}$$

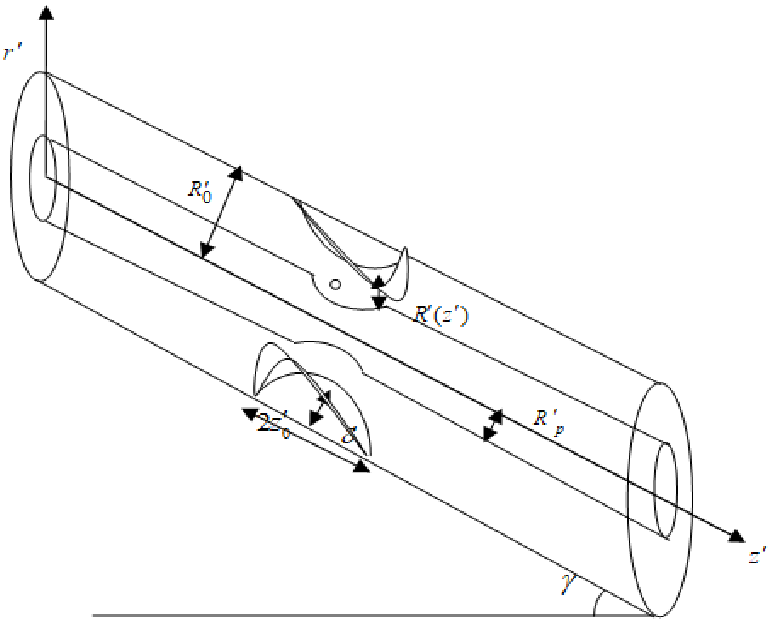

Fig1. Geometry of an inclined artery with axially non-symmetrical stenosis

Where A_0 and A_1 are the functions of z'. Pressure gradient in steady state, amplitude of the fluctuating component is denoted by the A_0 , A_1 respectively. ω'_{pa} denotes the frequency of oscillation of the pulsatile flow and its defined as $\omega'_{pa} = 2\pi f'_p$, with f'_p as the pulse rate frequency. Also, Radial velocity can be neglected due to its small magnitude.

Bingham Plastic fluid is defined by the following constitutive equation

$$\begin{cases} \tau'_s = \theta'_y + \mu'_0\left(-\dfrac{\partial v'_a}{\partial r'}\right); & \text{if } \tau'_s \geq \theta'_y \\ \dfrac{\partial v'_a}{\partial r'} = 0, & \text{if } \tau'_s < \theta'_y \end{cases} \tag{6}$$

Where, θ'_y denotes the yield stress and μ'_0 denotes the viscosity. The core region (i.e., $\tau'_s < \theta'_y$), shows the plug flow where the velocity gradient vanishes and fluid behavior can be observed in the region $\tau'_s \geq \theta'_y$.

BOUNDARY CONDITIONS

The boundary conditions are considered as

i) At $r' = 0$, τ'_s is finite (7)

ii) At $r' = R'(z')$, $v'_a = v'_{as}$ (8)

Where, v'_{as} denotes the axial slip velocity at the obstructed wall of the artery.

Now introducing the variables in the non-dimensional form as:

$$\left.\begin{array}{l} R(z) = R'(z')/R'_0, r = r'/R'_0, \; d = d'/R'_0, z = z'/R'_0, \; t = t'\omega'_{pa}, \; \omega = \omega'_{ba}/\omega'_{pa}, \\ F = A_0/4\rho' g, \delta = \delta'/R'_0, \; e = A_1/A_0, v_a = \dfrac{4v'_a \mu'_0}{A_0 R_0'^2}, \; v_{as} = \dfrac{4v'_{as}\mu'_0}{A_0 R_0'^2}, \tau_s = \dfrac{\tau'_s}{A_0 R'_0/2}, \\ \alpha^2 = \dfrac{R_0'^2 \omega'_{pa} \rho'}{\mu'_0}, B = a_0/A_0, \; \theta_y = \dfrac{\theta'_y}{A_0 R'_0/2} \end{array}\right\} \tag{9}$$

Where, α denotes the pulsatile Reynolds number and $A_0 R_0'^2/4\mu'_0$ represents the central line velocity in a Poiseuille flow.

With the help of non-dimensional variables, equation (2) can be written as

$$\alpha^2\left(\frac{\partial v_a}{\partial t}\right) = 4(1 + e\cos t) + 4B\cos(\omega t + \phi) - \left(\frac{2}{r}\right)\frac{\partial}{\partial r}(r\tau_s) + \frac{\sin\gamma}{F} \tag{10}$$

Non-dimensionalize Bingham Plastic fluid’s constitutive equation is defined by the following

$$\begin{cases} \tau_s = \theta_y + \dfrac{1}{2}\left(-\dfrac{\partial v_a}{\partial r}\right); & \text{if } \tau_s \ge \theta_y \\ \dfrac{\partial v_a}{\partial r} = 0, & \text{if } \tau_s < \theta_y \end{cases} \tag{11}$$

Now with the use of non-dimensional variables, the boundary conditions (7), (8) becomes

i) At $r = 0$, τ_s is finite (12)

ii) At $r = R(z)$, $v_a = v_{as}$ (13)

The geometry of an arterial stenosis in non-dimensional form is given by

$$\left.\begin{array}{ll} R(z) = 1 - \eta\left[L_s^{m-1}(z-d) - (z-d)^m\right], & d \le z \le d + L_s \\ \quad = 1, & \text{otherwise} \end{array}\right\} \tag{14}$$

Volumetric flow rate in the non-dimensional form is defined below

$$Q(z,t) = 4\int_0^{R(z)} v_a(z,r,t)\, r\, dr \tag{15}$$

Where $Q(z,t) = \dfrac{Q'(z',t')}{\pi A_0 (R'_0)^4/8\mu'_0}$; $Q'(z',t')$ denotes the volumetric flow rate.

Effective viscosity (μ'_e), is obtain as

$$\mu'_e = \pi\left(-\frac{\partial p'}{\partial z'}\right)(R'(z'))^4 / Q'(z',t')$$

With the help of non-dimensional variables, it’s defined as

$$\mu_e = R^4(1 + e\cos t)/Q(z,t) \tag{16}$$

3. METHOD OF SOLUTION

Now we expand equations (11) and (12) about α^2 as non-dimensionalize equations (11) and (12) have α^2 term which is time dependent. The axial velocity v_a, wall shear stress τ_s, plug core radius R_{pc}, plug core velocity v_{pc} and plug core shear stress τ_{pc} in terms of α^2 (where $\alpha^2 << 0$) are expressed as follows.

$$v_a(z,r,t) = v_{0a}(z,r,t) + \alpha^2 v_{1a}(z,r,t) + ... \quad (17)$$

$$\tau_s(z,r,t) = \tau_{0s}(z,r,t) + \alpha^2 \tau_{1s}(z,r,t) + ... \quad (18)$$

$$R_{pc}(z,t) = R_{0pc}(z,t) + \alpha^2 R_{1pc}(z,t) + ... \quad (19)$$

$$v_{pc}(z,t) = v_{0pc}(z,t) + \alpha^2 v_{1pc}(z,t) + ... \quad (20)$$

$$\tau_{pc}(z,t) = \tau_{0pc}(z,t) + \alpha^2 \tau_{1pc}(z,t) + ... \quad (21)$$

Substituting equations (17) and (18) in equation (10) and equating the α^2 terms and constant terms, we get

$$\frac{\partial}{\partial r}\left(r\tau_{0s}\right) = 2r\left[(1+\mathrm{e}\cos t)+B\cos(\omega t+\phi)+\frac{\sin\gamma}{4F}\right], \quad (22)$$

$$\frac{\partial v_{0a}}{\partial t} = -\frac{2}{r}\frac{\partial}{\partial r}\left(r\tau_{1s}\right) \quad (23)$$

Integrating equation (22) between 0 and R_{0pc}, with the help of boundary condition (12)

$$\tau_{0pc} = R_{0pc}K(t) \quad (24)$$

$$\text{Where } K(t) = \left[(1+\mathrm{e}\cos t)+B\cos(\omega t+\phi)+\frac{\sin\gamma}{4F}\right]$$

Integrating equation (22) between R_{0pc} and r and using equations (12) and (24), we get

$$\tau_{0s} = rK(t) \quad (25)$$

Substituting equations (17) and (18) in equation (11) we get

From Bingham model

$$\frac{\partial v_{0a}}{\partial r} = 2\left[\theta_y - \tau_{0s}\right] \quad (26)$$

$$\frac{\partial v_{1a}}{\partial r} = -2\tau_{1s} \quad (27)$$

Applying the expansion of v_a and τ_s from equation (17) and (18) in equation (12) and (13) we have

i) At $r = 0$, τ_{0s}, τ_{1s} are finite (28)

ii) At $r = R(z)$, $v_{0a} = v_{as}, v_{1a} = 0$ (29)

Integrating equation (26) between r and R, and by using equations (25) and (29), we have v_{0a} of Bingham model as follows

$$v_{0a} = v_{as} + K(t)\left(R^2 - r^2\right) - 2\theta_y(R-r) \quad (30)$$

From equation (30), first approximation v_{0pc} of plug core velocity of Bingham model, can be obtained as

$$v_{0pc} = v_{as} + K(t)\left(R^2 - R_{0pc}^2\right) - 2\theta_y (R - R_{0pc}) \quad (31)$$

Similarly solving equation (23) with the help of equations (30), (31) and boundary conditions (28), (29) we get the solutions for τ_{1s}, τ_{1pc} for Bingham model as

$$\tau_{1s} = \frac{rK'(t)(r^2 - 2R^2)}{8} \quad (32)$$

$$\tau_{1pc} = \frac{R_{0pc} K'(t)(R_{0pc}^2 - R^2)}{4}$$

(33)

From equation (27) and equation (32), axial velocity component v_{1a} and plug core velocity component v_{1pc} for Bingham model can be obtained as

$$v_{1a} = \frac{K'(t)}{4}\left[r^2 R^2 - \frac{(r^4 + 3R^4)}{4}\right] \quad (34)$$

$$v_{1pc} = \frac{K'(t)}{4}\left[R_{0pc}^2 R^2 - \frac{(R_{0pc}^4 + 3R^4)}{4}\right] \quad (35)$$

With the help of equation (30), (31) and (34), (35) in (17) and (20), the axial velocity v_a , plug core velocity v_{pc} for the Bingham model can be obtained as

$$v_a = v_{as} + K(t)\left(R^2 - r^2\right) - 2\theta_y (R - r) + \alpha^2 \frac{K'(t)}{4}\left[r^2 R^2 - \frac{(r^4 + 3R^4)}{4}\right] \quad (36)$$

$$v_{pc} = v_{as} + K(t)\left(R^2 - R_{0pc}^2\right) - 2\theta_y (R - R_{0pc}) + \alpha^2 \frac{K'(t)}{4}\left[R_{0pc}^2 R^2 - \frac{(R_{0pc}^4 + 3R^4)}{4}\right]$$

(37)

Neglecting α^2 terms and higher powers of α terms from equation (23), and with the use of equation (24), the R_{0pc} term of R_{pc} can be obtained as

$$r\big|_{\tau_{opc}=\theta_y} = R_{0pc} = \frac{\theta_y}{K(t)} \quad (38)$$

Using equations (24), (25) and (32), (33) in equations (18) and (21), the wall shear stress τ_s, τ_p for the Bingham model can be expressed as

$$\tau_s = RK(t) - \frac{\alpha^2 K'(t) R^3}{8} \quad (39)$$

$$\tau_{pc} = R_{0pc} K(t) + \frac{\alpha^2 R_{0pc} K'(t)(R_{0pc}^2 - R^2)}{4} \quad (40)$$

With the help of equation (17) and expression of axial velocity of Bingham Plastic fluid, we get volumetric flow rate Q_b for Bingham model as

$$Q_b = 2R^2 v_{as} - \frac{4}{3}\theta_y \left(R^3 - R_{0pc}^3\right) + \left(R^4 - R_{0pc}^4\right) K(t) - \frac{\alpha^2 K'(t)}{12}\left\{(R_{0pc}^6 + 2R^6) - 3R_{0pc}^4 R^2\right\} \quad (41)$$

Neglecting higher powers of α terms and α^4 terms from the equation (19), the R_{1pc} approximation of plug core radius can be obtained as

$$R_{1pc} = \frac{\tau_1\left(R_{0pc}\right)}{K(t)} \tag{42}$$

With the help of equations (19), (38) and (42), the R_{pc} for Bingham model can be expressed as

$$R_{pc} = \frac{\theta_y}{K(t)} + \frac{\alpha^2 R_{0pc} K'(t)(R_{0pc}^2 - R^2)}{4} \tag{43}$$

In dimensionless form the effective viscosity for Bingham model, with the help of equation (16) and (41) can be obtained

$$\mu_b = \left(R(z)\right)^2 (1 + e\cos t)\left\{Q_b\right\}^{-1} \tag{44}$$

4. NUMERICAL RESULTS AND DISCUSSION

Numerical computations of these quantities for major physiological significance have been done with the help of MATLAB.

4.1 AXIAL VELOCITY

Fig. 2 reveals the effect of slip velocity and inclination on axial velocity versus radial distance for fixed values F = 0.2, α = 0.2, ϕ = 0.2, ω = 1, e = 1, m = 2, t = 1, δ = 0.1. we observed that v_{as} and γ augmented the axial velocity of fluid in steadily manner. Fig. 3 depicts the variation of δ on axial velocity with radial distance for fixed parameters B = 1, F = 0.2, α = 0.2, e = 1, ω = 1, m = 2, t = 1, v_{as} = 0.1. Fig. 3 reveals that velocity diminishes as stenosis height increases.

4.2 PLUG CORE VELOCITY

Fig.4 illustrates the body acceleration effect on plug core velocity versus time for Bingham fluid for given parametric values $\gamma = 30^0$, F = 0.2, ϕ = 0.2, e = 0.05, δ = 0.1, ω = 1. Fig. 4 shows that in core region velocity of Bingham fluid increases as body acceleration increases. It also describes that plug core velocity enhances as body acceleration decreases from time 0^0 to 180^0 and then increases from 180^0 to 360^0.

4.3 VOLUMETRIC FLOW RATE

Fig.5 reveals the variation of volumetric flow rate with stenosis height for different values of yield stress and body acceleration for Bingham Plastic fluid and with values $\gamma = 30^0$, F = 0.2, ϕ = 0.2, e = 0.05, δ = 0.1, ω = 1, v_{as} = 0.1. It has been observed in Fig.5 that as yield stress and stenosis height increases flow rate decreases, this happens because lumen size reduces. Also, presence of body acceleration increases the flow rate as plug region contracts on increasing the body acceleration, and hence more flow takes place in the tube.

4.4 WALL SHEAR STRESS

Fig.6 depicted the variation of wall shear stress with time for Bingham Plastic fluid and for different values of m and B with $\gamma = 30^0$, F = 0.2, ϕ = 0.2, e = 0.05, δ = 0.1, ω = 1. From Fig. 6, it has been found that wall shear stress is more in the presence of body acceleration as compared to the absence case, i.e. for zero value of body acceleration and decreases partially with time 180^0 towards a minimum point.

4.5 EFFECTIVE VISCOSITY

The effect of stenosis height and yield stress has been observed for effective viscosity versus slip velocity in Fig. 7 and it described that the effective viscosity increases with the increase of stenosis height and yield stress whereas decreases for hiked slip velocity.

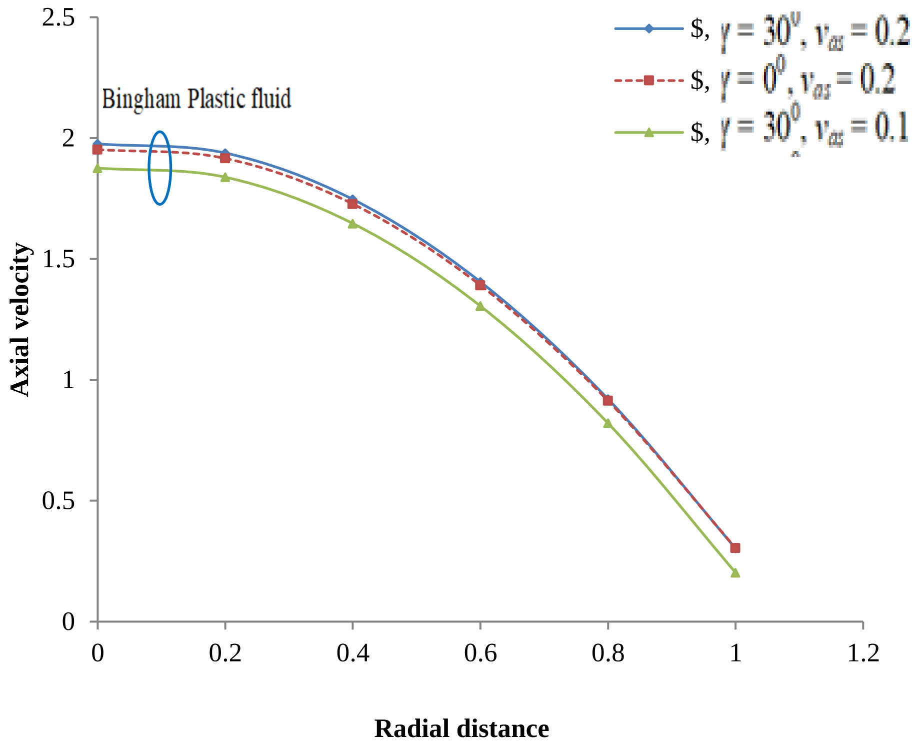

Fig.2 Variation of axial velocity with the radial distance for different values of inclined angle and slip, $ shows Bingham Plastic fluid.

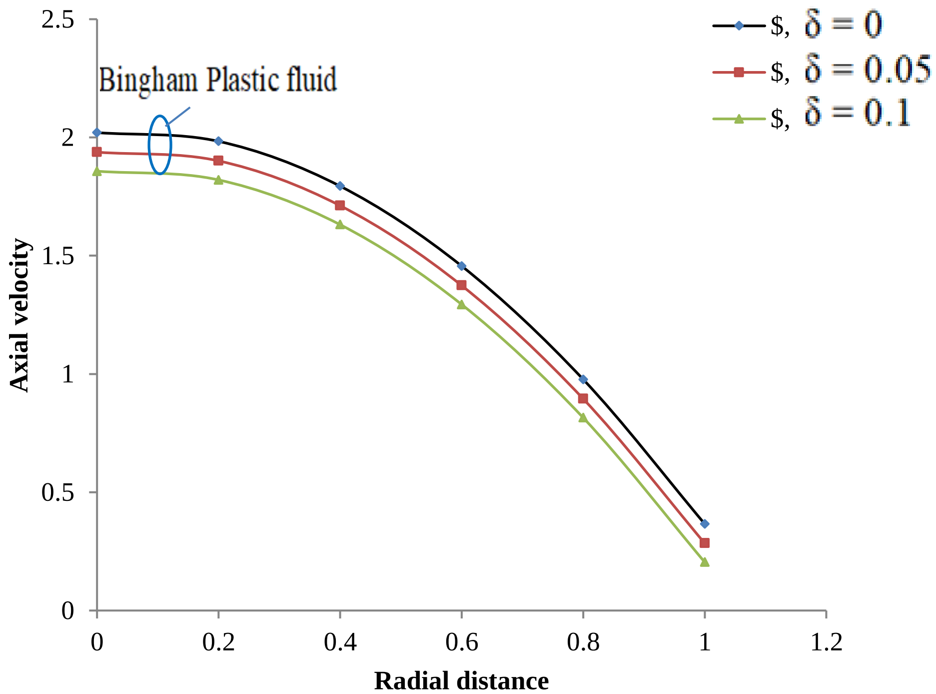

Fig. 3 Variation of axial velocity with the radial distance for different values of stenosis height, $ shows Bingham Plastic fluid.

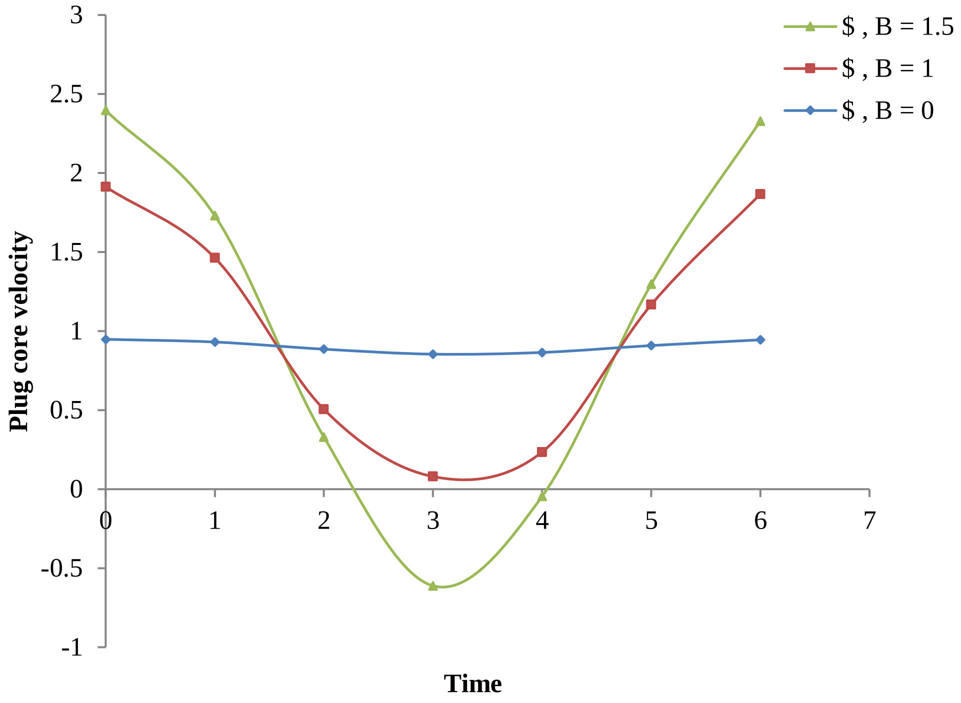

Fig. 4 Variation of plug core velocity with time for different values of body acceleration , $ shows Bingham Plastic fluid.

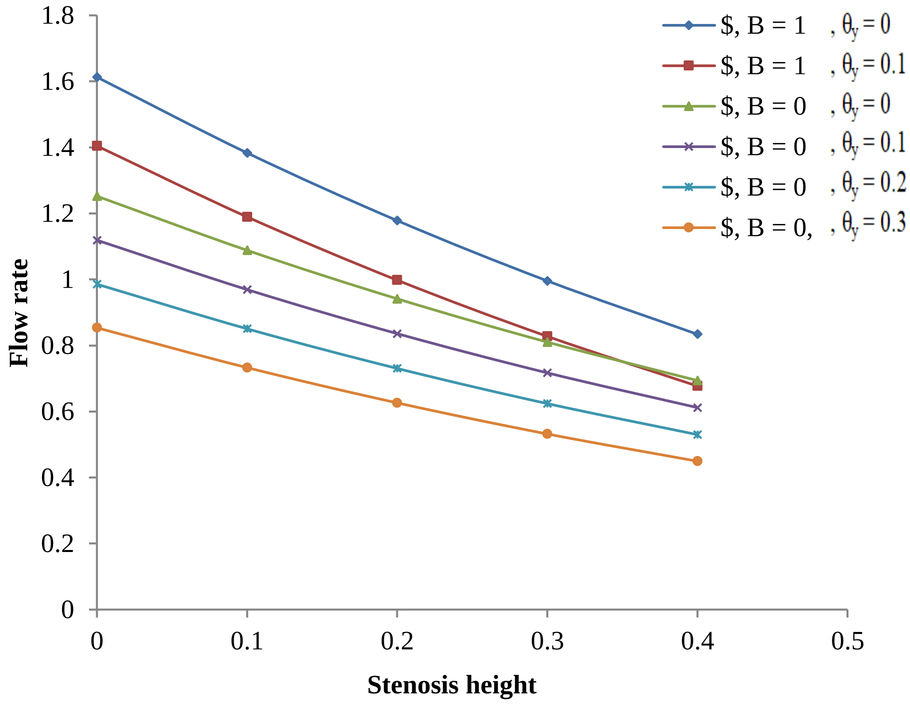

Fig. 5 Variation of flow rate with stenosis height for different values of yield stress and body acceleration, $ shows Bingham Plastic fluid.

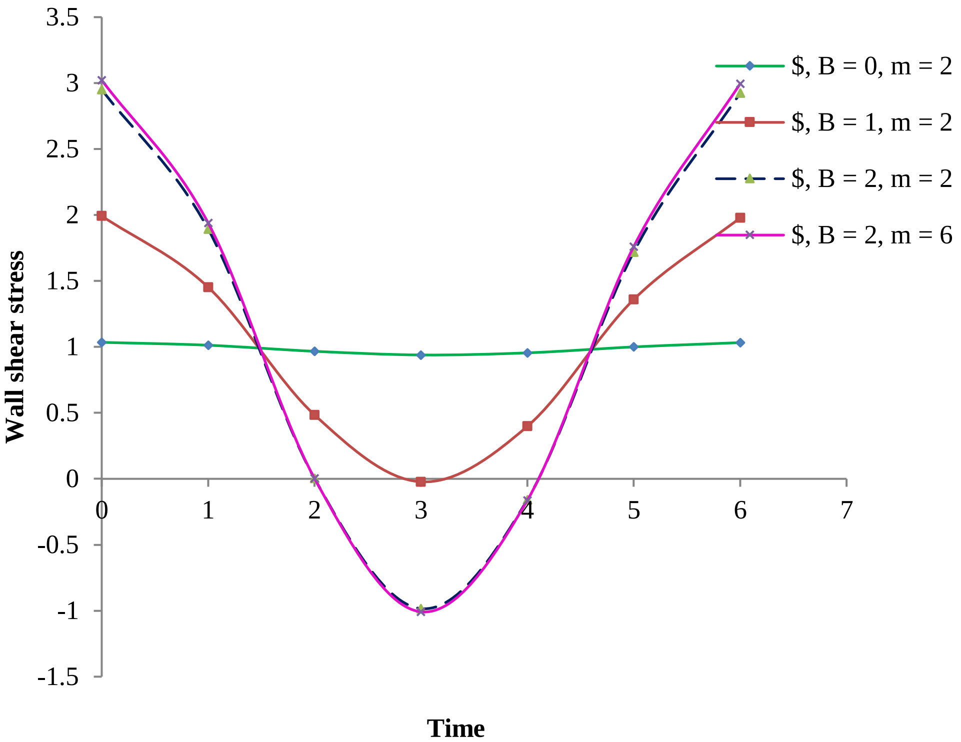

Fig. 6 Variation of wall shear stress with time for different values of shape parameter and body acceleration , $ shows Bingham Plastic fluid.

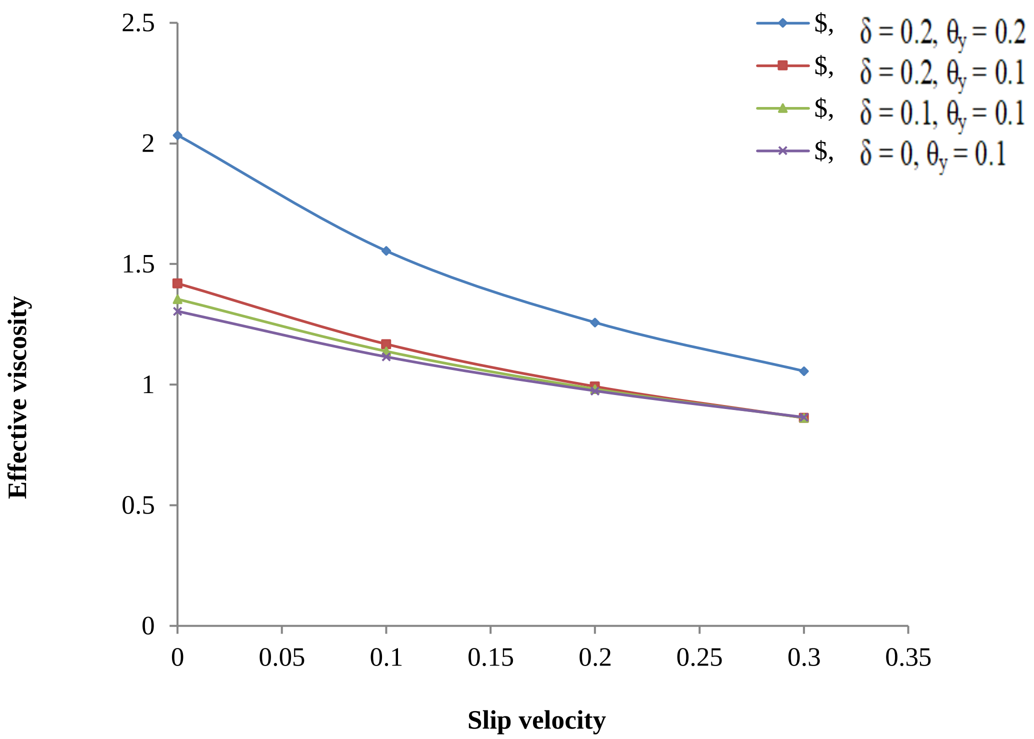

Fig.7 Variation of effective viscosity with slip velocity for different values of yield stress and stenosis height, $ shows Bingham Plastic fluid.

5. CONCLUSION

Here we observed that velocity augmented as stenosis loses their shape from $m = 2$ to $m = 10$. Plug core radius diminishes as body acceleration and inclination increases. Flow rate increases as body acceleration increases and decreases as yield stress and stenosis height increases. It has been found that due to increase of body acceleration which created more friction inside the artery, wall shear stress increases.

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